1. (25 points)
   a. For \( f(x, y) = x + e^{xy} \), find
      \[
      \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \text{ and } \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}, \frac{\partial^2 f}{\partial x \partial y}.
      \]
   b. For \( g(x, y, z) = xyz + xy \), find
      \[
      \frac{\partial^2 g}{\partial x \partial z}(2, 2, 1).
      \]

2. (20 points)
   Use Lagrange multipliers to find the values of \( x \) and \( y \) that minimize the function
   \[-x^2 - 3xy - \frac{1}{2}y^2 + 10\]
   subject to the constraint \( 30 - x - y = 0 \).

3. (20 points)
   Use the method of least squares to find a line that best fits the points \((-1, -1), (0, 0)\) and \((2, 1)\). Use this to estimate the value of \( y \) when \( x = 1 \).

4. (25 points)
   Let \( f(x, y) = xy - \frac{1}{3}x^3 - \frac{1}{2}y^2 \).
   a. Find all points \((x, y)\) where \( f(x, y) \) has a possible relative maximum or minimum.
   b. Use the second derivative test to determine if these points give rise to relative maxima, minima or saddles. If the second derivative test is inconclusive, state so.

5. (10 points)
   Suppose that the function \( P(x, y) \) gives the US demand for OPEC oil, where \( x \) is the price of OPEC oil and \( y \) is the price of North Shore oil. Would you expect \( \frac{\partial P}{\partial x} \) to be positive or negative? Would you expect \( \frac{\partial P}{\partial y} \) to be positive or negative? Explain why.