Math 231 Spring 2007

Practice Final Solutions

Show all work.

1. (8 points) Use the method of least squares to find the straight line that best fits the points 
     \[ (2, 2), \quad (3, 0), \quad (7, -1). \]

2. (10 points) Evaluate 
     \[ \int \int_R (x^2 y + y^2) \, dxdy \]
     where \( R \) is the rectangle \( 0 \leq x \leq 2 \) and \( 2 \leq y \leq 3 \).

3. (12 points) Find the critical points of 
     \[ f(x, y) = x^3 - y^2 - 3x + 4y, \]
     and determine whether they are minima, maxima or saddle points.

4. (10 points) (a) Sum the series 
     \[ \frac{3}{4} - \frac{3}{16} + \frac{3}{64} - \frac{3}{256} + \cdots. \]

    (b) A doctor wishes to give a patient a daily dose of \( D \) mgs of a certain medicine. The doctor knows that the body eliminates each day 70% of the amount present in the body. In the long run, the doctor would like the amount of drug in the body to approach 6 mgs. What should \( D \) be?

5. (8 points) Find the third Taylor polynomial at \( x = 1 \) for the function 
     \[ f(x) = \frac{1}{(x + 1)} = (x + 1)^{-1}. \]

6. (12 points) Use separation of variables to solve the following differential equations:

    (a) \( \frac{dy}{dt} = yt, \quad y(0) = 1; \) \quad (b) \( \frac{dy}{dt} = y^3t + y^3, \quad y(0) = 1. \)

7. (8 points) If a radioactive substance decays from 5 grams to 3 grams in 6 days, when will only 1 gram remain?

8. (10 points) For the differential equation \( \frac{dy}{dt} = (y + 1)(y - 7) \) sketch the constant solutions and the solutions corresponding to \( y(0) = -3, \quad y(0) = 2, \quad y(0) = 10. \)

9. (10 points) A scientist is growing a bacteria culture. She knows that the bacteria grow at a rate of 3% per day. In addition, she wants to remove \( M \) bacteria each day. Initially she has a culture of 500,000 bacteria.

    (a) Find a differential equation for \( y(t) \), the population size after \( t \) days.
(b) What is the maximum number of bacteria she can remove each day without eventually killing off the population of bacteria?

10. **(6 points)** Use two iterations of the Newton-Raphson method to approximate a zero of \( x^3 - x - 2 \) between 1 and 2. Let \( x_0 = 1 \).

11. **(6 points)** Use Euler's method with \( n = 2 \) to approximate \( y(2) \) where \( y(t) \) is a solution of \( \frac{dy}{dt} = t - 2y \) and \( y(0) = 2 \).