Assessed Problem Set 3

Please answer all questions and hand your answers in to me by December 3, 2004. In question 1, you are required to use MATLAB GMM GUI discussed in the computer labs; please hand in a hard copy of your program for calculating the moment and its derivative along with your answers to the questions.

1. In Assessed Problem Set 2, you estimated the CBAPM using both VWR and EWR. In this question, you must test certain hypotheses related to this model. In each case, the basic moment condition is the same as before, that is

\[ E[u_t(\theta_0) \otimes z_t] = 0 \]  \hspace{1cm} (1)

where

\[ u_t(\theta) = \begin{bmatrix} \delta x_{1,t+1}^{\gamma-1}x_{2,t+1} - 1 \\ \delta x_{1,t+1}^{\gamma-1}x_{3,t+1} - 1 \end{bmatrix} \]

where \( \theta = (\delta, \gamma)' \), \( x_{1,t+1} = c_{t+1}/c_t \), \( x_{2,t+1} \) denotes EWR, \( x_{3,t+1} \) denotes VWR, and \( z_t \) is the vector of instruments. Set \( z_t = (1, x_{1,t}, x_{2,t}, x_{3,t})' \).

(i) Test whether the data are consistent with the moment condition in (1).

(ii) Test the null hypothesis (1) holds versus the alternative hypothesis,

\[ E[u_{1,t}(\theta_0)z_t] = 0 \text{ but } E[u_{2,t}(\theta_0)z_t] \neq 0 \]  \hspace{1cm} (2)

where

\[ \begin{bmatrix} u_{1,t}(\theta) \\ u_{2,t}(\theta) \end{bmatrix} = \begin{bmatrix} \delta x_{1,t+1}^{\gamma-1}x_{2,t+1} - 1 \\ \delta x_{1,t+1}^{\gamma-1}x_{3,t+1} - 1 \end{bmatrix} \]

(iii) Test the hypothesis that the population moment condition in (1) is structurally stable using the fixed break point date of October, 1979 (the date of the change in the operating procedures of the Federal Reserve Board). Do these results give any indication about the form of the instability?

2. In class we discussed the limiting distribution of the overidentifying restrictions test under the null hypothesis that the model is correctly specified. In this question you consider the limiting distribution of the first step minimand. Let

\[ A_T = T g_T(\hat{\theta}_T)' W_T g_T(\hat{\theta}_T) \]

where \( \hat{\theta}_T = \arg\min_{\theta \in \Theta} g_T(\theta)' W_T g_T(\hat{\theta}_T) \).
(i) Show that under the condition of Theorem 3.3

\[ A_T \xrightarrow{d} \sum_{i=1}^{q-p} \lambda_i n_{q,i}^2 \]

where \( \lambda_i \) are the nonzero eigenvalues of \( BB' = S \), \( P(\theta_0) \) is the projection matrix defined in Theorem 3.3, \( n_{q,i} \) is the \( i^{th} \) element on \( n_q \), a \( q \times 1 \) with distribution \( N(0, I_q) \).

(ii) Verify that this result is identical to the result in Theorem 5.1 if \( W = S^{-1} \).

3. In this question, you show that Neyman and Pearson’s goodness of fit statistic can be viewed as an overidentifying restrictions test. The framework and notation is the same as in Section 1.2 of the text. Suppose estimation of the \( p \times 1 \) parameter vector \( \theta_0 \) is to be based on the population moment condition,

\[ E[f(v_t, \theta_0)] = 0 \]  

where

\[ f(v_t, \theta_0) = \begin{bmatrix} D_1(1) - h(1; \theta_0) \\ D_2(1) - h(2; \theta_0) \\ \vdots \\ D_k(k) - h(k; \theta_0) \end{bmatrix} \]

where \( k > p, v_t = (D_1(1), D_2(1), \ldots, D_k(k))^T \), and \( D_k(i) \) is an indicator variable which takes the value one if the \( k^{th} \) outcome of the experiment lies in the \( i^{th} \) group and takes the value zero otherwise for \( \{i = 1, 2, \ldots, k; t = 1, 2, \ldots, T\} \). Assume that \( v_t \) is iid, (3) holds and \( h(i; \theta_0) > 0 \) for all \( i \).

(i) Assuming that \( T^{-1/2} \sum_{t=1}^{T} f(v_t, \theta_0) \xrightarrow{d} N(0, S) \), show that if (3) holds then

\[ S = \begin{bmatrix} h_1(1 - h_1) & -h_1h_2 & -h_1h_3 & \cdots & -h_1h_k \\ -h_1h_2 & h_2(1 - h_2) & -h_2h_3 & \cdots & -h_2h_k \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ -h_1h_k & -h_2h_k & -h_3h_k & \cdots & h_k(1 - h_k) \end{bmatrix} \]

where \( h_i = h(i; \theta_0) \) for \( i = 1, 2, \ldots, k \).

(ii) Show that \( S \) is singular.

(iii) The singularity can be circumvented by dropping one class, the \( k^{th} \) class say. Let \( f_1(v_t, \theta_0) \) be the \( (k - 1) \times 1 \) matrix whose \( i^{th} \) element is the corresponding element of \( f(v_t, \theta_0) \). It follows from part (i) that \( T^{-1/2} \sum_{t=1}^{T} f_1(v_t, \theta_0) \xrightarrow{d} N(0, S_1) \) where

\[ S_1 = \begin{bmatrix} h_1(1 - h_1) & -h_1h_2 & -h_1h_3 & \cdots & -h_1h_{k-1} \\ -h_1h_2 & h_2(1 - h_2) & -h_2h_3 & \cdots & -h_2h_{k-1} \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ -h_1h_{k-1} & -h_2h_{k-1} & -h_3h_{k-1} & \cdots & h_{k-1}(1 - h_{k-1}) \end{bmatrix} \]
Show that $S^{-1}_1 = S$ where

$$S = \begin{bmatrix} \frac{1}{h_1} + \frac{1}{h_k}, & \frac{1}{h_1} + \frac{1}{h_k}, & \ldots & \frac{1}{h_1} + \frac{1}{h_k} \\ \frac{1}{h_2}, & \frac{1}{h_2}, & \ldots & \frac{1}{h_2} \\ \frac{1}{h_3}, & \frac{1}{h_3}, & \ldots & \frac{1}{h_3} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{h_k}, & \frac{1}{h_k}, & \ldots & \frac{1}{h_k} \end{bmatrix}$$  \hspace{1cm} (7)

(iv) Show that Neyman and Pearson’s minimum chi-squared minimand for this problem,

$$GF_T(\theta_0) = T \sum_{i=1}^k \frac{\hat{p}_i - h(i; \theta_0)^2}{p_i}$$  \hspace{1cm} (8)

is proportional to the minimand of the GMM estimator based on $E[f_1(v_t, \theta_0)] = 0$ with weighting matrix $W_T = \hat{S}$ where

$$\hat{S} = \begin{bmatrix} \frac{1}{\hat{p}_1} + \frac{1}{\hat{p}_k}, & \frac{1}{\hat{p}_1} + \frac{1}{\hat{p}_k}, & \ldots & \frac{1}{\hat{p}_1} + \frac{1}{\hat{p}_k} \\ \frac{1}{\hat{p}_2}, & \frac{1}{\hat{p}_2}, & \ldots & \frac{1}{\hat{p}_2} \\ \frac{1}{\hat{p}_3}, & \frac{1}{\hat{p}_3}, & \ldots & \frac{1}{\hat{p}_3} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\hat{p}_k}, & \frac{1}{\hat{p}_k}, & \ldots & \frac{1}{\hat{p}_k} \end{bmatrix}$$

and $\hat{p}_i = T^{-1} \sum_{t=1}^T D_t(i) > 0$ for all $i$.

(v) Let $\tilde{\theta}_T = \arg\min_{\theta \in \Theta} GF_T(\theta)$. Show that $GF_T(\tilde{\theta}_T)$ is asymptotically equivalent to the overidentifying restrictions test based on $E[f_1(v_t, \theta_0)] = 0$. 

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