Assessed Problem Set 1

Please answer all questions and hand your answers in to me in class on Monday, September 27, 2004. Question 4 requires you to write a program in MATLAB; please hand in a hard copy of your program.

1. Suppose demand and supply for hogs are generated by the system

\[
\begin{align*}
q_t^D &= \alpha_{0,1} n_{1,t} + \alpha_{0,2} n_{2,t} + \alpha_{0,3} p_t + u_t^D \\
q_t^S &= \beta_0 p_t + u_t^S \\
q_t^D &= q_t^S = q_t
\end{align*}
\]

(1)

where \(q_t^D, q_t^S\) represent demand and supply in year \(t\), \(p_t\) is the price of hogs in that year, \(n_{1,t}\) is the price of beef and \(n_{2,t}\) is per capita consumption expenditure. The market is assumed to clear and the total quantity produced is denoted \(q_t\). For the purposes of this discussion it is assumed that \(E[u_t^D|\Omega_t] = E[u_t^S|\Omega_t] = 0\) where \(\Omega_t\) is the information set at time \(t\), and that \(n_t = (n_{1,t}, n_{2,t}) \in \Omega_t\). Assume that: \(\alpha_{0,3} \neq \beta_0\) and \(\alpha_{0,i} \neq 0\) for \(i = 1, 2, 3\).

(a) Let \(x_t = (n_{1,t}, n_{2,t}, p_t)'\) and \(\alpha_0 = (\alpha_{0,1}, \alpha_{0,2}, \alpha_{0,3})'\). Suppose that the parameters of the demand equation, \(\alpha_0\) are to be estimated by GMM based on the following moment condition,

\[
E[z_t(q_t - \alpha_0 x_t)] = 0
\]

where \(z_t\) is a vector of instruments with \(z_t \in \Omega_t\). Show that \(\alpha_0\) is unidentified.

(b) Suppose that the parameters of the supply equation, \(\beta_0\) are to be estimated by GMM based on the following moment condition,

\[
E[z_t(q_t - \beta_0 p_t)] = 0
\]

where \(z_t\) is a vector of instruments. Show that \(\beta_0\) is identified in this model if \(z_t\) contains variables that are correlated with \(n_{1,t}\) and/or \(n_{2,t}\).

2. Let \(\{v_t\}\) be a sequence of random variables such that \(E[v_t] = \mu\).

(a) Show that \(\sup_t E|v_t^2| < \infty\) implies \(\sup_t \sup_i |Cov(v_t, v_{t-i})| < \infty\).

(b) Show that

\[
Var[\bar{v}_T] = T^{-2} \left\{ \sum_{t=1}^{T} \sigma_t^2 + 2 \sum_{m=1}^{T-1} \sum_{t=m+1}^{T} \sigma_{t,t-m} \right\}
\]

where \(\sigma_{t,t-i} = Cov(v_t, v_{t-i})\).
(c) Show that if \( T^{-1} \sum_{m=1}^{T} b_m \to 0 \) then \( T^{-1} \sum_{t=1}^{T} v_t \xrightarrow{p} \mu \) where \( b_m = \sup_t |\sigma_{t,t-m}| \) and \( \sup_t E|v_t^2| < \infty \).

3. Let \( \{v_t, t = -\infty, \ldots, -1, 0, 1, \ldots \infty\} \) be generated via
\[
v_t = \sum_{i=0}^{\infty} \psi_i e_{t-i}
\]
where \( \{e_t\}_{t=-\infty}^{\infty} \) is a sequence of iid random variables with \( E[e_t] = 0 \) and \( E[e_t^2] < \infty \), and \( \sum_{i=0}^{\infty} |\psi_i| < \infty \).

(a) Show that \( V_T = T^{-1/2} \sum_{t=1}^{T} v_t \) can be decomposed as follows:
\[
V_T = Y_{T,k} + R_{T,k}
\]
where \( Y_{T,k} = T^{-1/2} \sum_{t=1}^{T} y_{t,k}, R_{T,k} = T^{-1/2} \sum_{t=1}^{T} r_{t,k}, y_{t,k} = \sum_{i=0}^{k} \psi_i e_{t-i}, r_{t,k} = \sum_{i=k+1}^{\infty} \psi_i e_{t-i} \).

(b) Show that \( y_{T,k} \) is a \( k \)-dependent process.

(c) Show that \( T^{1/2}(\hat{\theta}_T - \theta) \xrightarrow{d} N(0, S) \) as \( T \to \infty \) where \( S = \gamma_0 + 2 \sum_{i=1}^{\infty} \gamma_i \).

4. In this question, you must write a program in MATLAB to estimate the parameter of the supply equation for pork in (1) by IV using annual data for the US over the period 1970-1994. A data file has been mailed to your computer account which contains the following six variables:

- **YR**: the year.
- **EXP**: per capita consumption expenditure.
- **BEEFQ**: per capita beef production in pounds per person.
- **PORKQ**: per capita pork production in pounds per person.
- **PORKP**: price of pork in cents per pound (constant 1982 $).
- **BEEFP**: price of pork in cents per pound (constant 1982 $).

Use all data in logs for the estimation.

Write a program in MATLAB to calculate the following.

(a) Calculate the GMM estimator of \( \beta_0 \) based on (3) using two choices of \( z_t \): (i) **EXP**; (ii) **BEEFP**. Contrast the estimates and the associated 95% confidence intervals for \( \beta_0 \).

(b) Calculate the two step GMM estimators of \( \beta_0 \) based (3) using \( z_t \) set equal to \( (\text{EXP},\text{BEEFP})' \) with two different choices of first step weighting matrix: (i) \( W_T = I_2 \); (ii) \( W_T = (T^{-1}Z'Z)^{-1} \). Contrast the estimates and the associated 95% confidence intervals for \( \beta_0 \).

(c) Calculate the overidentifying restrictions test statistic. Does the model appear consistent with the data?