Solutions to exercises in MATLAB Handout # 1

As on the handout, MATLAB commands are in bold type.

1.-3. Obvious from screen.

4. The calculations are executed as follows:
   \[ h = \text{ones}(5,1); \]
   \[ H = \text{eye}(5) - h \ast \text{inv}(h' \ast h) \ast h'; \]
   \[ H - H' \]
   \[ H \ast H - H \]
   At first glance the last calculation does not appear to yield zero, but note that the matrix is scaled by \(1.0 \times 10^{-15}\). So the result is effectively zero. This is due to rounding in the \(H^2\) calculation.

5. Given the structure of \(H\), it has rank equal to four. Therefore, as it is idempotent, it has four eigenvalues of one and one of zero. The eigenvalues can be calculated as follows: \(\text{eig}(H)\).

6. The trace equals the sum of the eigenvalues which in this case is four. The trace is calculated as follows: \(\text{trace}(H)\).

7. Notice that the reinitialization of the seed in (b) means that C and D are equal in case (b) but not in case (a).

8. \(5 \ast \text{ones}(3,2) + 2 \ast \text{randn}(3,2)\).

9. A matrix of the dimension of \(A\) whose elements are random draws from a standard normal distribution.

10. Recall that the orthogonal matrix of eigenvectors can be used to diagonalize \(A\) and so the calculations are as follows. \(A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}; [F, L] = \text{eig}(A); F' \ast AF\). Note that \(F'AF = L\).

11. Create the function m-file LS.m as follows:
   \[ x = \text{LS}(A, b); x = \text{inv}(A' \ast A) \ast A' \ast b; \]
   To verify that the least squares error is in the left null space of \(A\), the calculation is: \((A \ast x - b) \ast A\). Notice again that the answer is not exactly zero due to rounding but is obviously effectively zero.