Assessed Computer Assignment 1

Answer all questions. Your answers must be handed in to me in class on Monday, February 21. Please hand in a hard copy of both your program and all relevant output.

The data series for this assignment is the natural logarithm of seasonally adjusted quarterly consumption for the US from 1947 quarter 1 through 2002 quarter 4. The series can be downloaded from the course web page: http://www4.ncsu.edu/~arhall/ECG752.htm.

Let $x_t$ denote the natural logarithm of quarterly consumption and $v_t$ denote its first difference, that is $v_t = x_t - x_{t-1}$.

1. Calculate the autocorrelation and partial autocorrelation functions of $x_t$ up to lag 20. Do these functions look like those of a stationary invertible ARMA$(p, q)$ process? Briefly justify your answer.

2. Calculate the autocorrelation and partial autocorrelation functions of $v_t$ up to lag 20. Do these functions look like those of a stationary invertible ARMA$(p, q)$ process? If so, then what values of $p$ and $q$ do these functions suggest? Briefly justify your answer.

3. Estimate an AR(1) model for $v_t$ and summarize the results. Does the model appear well specified? Briefly justify your answer.

4. Estimate an AR(8) model for $v_t$ and summarize the results. Does the model appear well specified? Briefly justify your answer.

5. Estimate a MA(1) model for $v_t$ and summarize the results. Does the model appear well specified? Briefly justify your answer.

6. Estimate a MA(8) model for $v_t$ and summarize the results. Does the model appear well specified? Briefly justify your answer.

7. Estimate an ARMA(3,1) model for $v_t$ and summarize the results. Does the model appear well specified? Briefly justify your answer.

One popular approach to order estimation is the use of information criteria. A number of criteria have been proposed. Here we concentrate on the two most common: Akaike’s information criterion (AIC) and Schwarz’s information criterion (SIC - and sometimes BIC with B for Bayesian). Both have a similar structure: the estimated ARMA orders are the values of $p$ and $q$ which minimize a
specified criterion function over the set of all possible lag lengths. The generic structure for the criteria is:

\[ T\ln|\hat{\sigma}_T^2(p, q)| + C_T(p, q) \]

where \( \hat{\sigma}_T^2(p, q) \) is the estimated variance for \( w_t \), the white noise innovation in the ARMA\((p, q)\) model, for AIC, \( C_T(p) = 2(p + q) \) and for SIC, \( C_T(p) = (p + q)\ln|T| \). Therefore with AIC, the estimated order is

\[(\hat{p}_T, \hat{q}_T) = \arg\min_{(p,q) \in \mathcal{K}} \left\{ T\ln|\hat{\sigma}_T^2(p, q)| + 2(p + q) \right\}\]

and with SIC, the estimated order is

\[(\hat{p}_T, \hat{q}_T) = \arg\min_{(p,q) \in \mathcal{K}} \left\{ T\ln|\hat{\sigma}_T^2(p, q)| + (p + q)\ln|T| \right\}\]

where \( \mathcal{K} \) is the set of all possible combinations for \( (p, q) \). It can be seen that these information criteria involve a trade-off between the residual sum of squares, as captured by \( T\ln|\hat{\sigma}_T^2(p, q)| \), and a deterministic penalty term involving the number of parameters \( p + q \). Note that these criteria are presented as part of the output for proc arima in which AIC is AIC and SIC is SBC.

8. Let \( \mathcal{K} = \{(1, 0), (8, 0), (0, 1), (0, 8), (3, 1)\} \) and use: (a) SIC to determine the appropriate order for the ARMA model for \( v_t \); (b) use AIC to determine the appropriate order for the ARMA model for \( v_t \).

9. Forecast \( v_t \) for 2003Q1, 2003Q2, 2003Q3, 2003Q4 using: (a) the estimated model for the choices of \( (p, q) \) determined in Question 8.(a); (b) the estimated model for the choices of \( (p, q) \) determined in Question 8.(b).

10. The actual figures for \( v_t \) in 2003Q1 through 2003Q4 are: 0.0297, -0.0055, 0.0281 and 0.0053 respectively. One way to measure forecasts accuracy is to calculated the forecast root mean square error:

\[ RMSE = \sqrt{\sum_{s=1}^{n} (\hat{v}_{t+s,t} - v_{t+s})^2} \]

where \( \hat{v}_{t+s,t} \) is the forecast of \( v_{t+s} \) given \( v_t, v_{t-1}, \ldots, v_1 \). Calculate \( RMSE \) for the forecasts generated in Question 9 parts (a) and (b). Which model yields more accurate forecasts?