MA 534 - Homework #5
due on 11/15/12

1. Solve the following problems from the textbook: 6.2, 6.6.

2. Consider the following eigenvalue problem:

\[-X''(x) = \lambda X(x), \quad 0 < x < L,\]
\[X'(0) - a_0 X(0) = 0,\]
\[X'(L) + a_l X(L) = 0.\]

(a) Show that \(\lambda = 0\) is an eigenvalue if and only if
\(a_0 + a_l = -a_0 a_l L.\)
(b) Find the eigenfunction corresponding to the zero eigenvalue.
(c) Show that if \(a_0 = a_l = a\) then there are no negative eigenvalues if \(a \geq 0\), there
one if \(-2/L < a < 0\), and there are two if \(a < -2/L\).

3. Consider any series of functions on any finite interval. Show that if the series converges
uniformly, then it also converges in the \(L^2\) sense and pointwise.

4. Find the Fourier series of the function

\[f(x) = \begin{cases} 0, & \text{if } -1 \leq x < 1/2 \\ 1, & \text{if } 1/2 \leq x \leq 3/4 \\ 2, & \text{if } 3/4 < x \leq 1 \end{cases}\]

and determine the sum of the Fourier series on the interval.

5. Let

\[f(x) = \begin{cases} -1 - x, & \text{if } -1 < x < 0 \\ 1 - x, & \text{if } 0 < x < 1 \end{cases}\]

(a) Find the Fourier series of \(f(x)\) in the interval \((-1, 1)\).
(b) Does it converge in the \(L^2\) sense?
(c) Does it converge pointwise?
(d) Does it converge uniformly?

6. Let \(f(x) = x^2\) for \(0 \leq x \leq 1\).

(a) Calculate its Fourier sine series. Plot several partial sums to illustrate the conver-
gence of the Fourier series.
(b) Calculate its Fourier cosine series. Plot several partial sums to illustrate the
convergence of the Fourier series.
(c) Use the obtained result and Parseval’s identity to show that

\[\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}.\]