Differential Equation Model.

Newton's law of motion then gives the differential equation

\[ m \, u_t = -mg + k \, u^2. \]

As time evolves, the speed will approach \( u = (mg/k)^{1/2} \).
Method of Solution.

Euler's Numerical Method for Approximating $y' = g(t,y)$ and $y(0)$ given.

$$Y(i+1) = Y(i) + dt * g(i*dt, Y(i)).$$

Improved Euler's Numerical Method for Approximating $y' = g(t,y)$ and $y(0)$ given.

$$Y(i+1) = Y(i) + dt * g(i*dt, Y(i))$$

$$Y(i+1) = Y(i) + dt/2 * (g(i*dt, Y(i)) + g((i+1)*dt, Y(i+1))).$$
<table>
<thead>
<tr>
<th>KK</th>
<th>erreul</th>
<th>errimerr</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>3.1152</td>
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<td>16</td>
<td>1.5347</td>
<td>0.0326</td>
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<tr>
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<tr>
<td>64</td>
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<td>0.0020</td>
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</table>
Matlab Implementation.

```matlab
function fmass = fmass(t,y)
    fmass = -32 + .1*abs(y);

% your name, your student number, lesson number
clear;
y(1) = 0.;
T = 100.;
KK = 100
h = T/KK;
t(1) = 0.;
for k = 1:KK
    y(k+1) = y(k) + h*fmass(t(k),y(k));
t(k+1) = t(k) + h;
y(k+1) = y(k) + .5*h*(fmass(t(k),y(k)) + fmass(t(k+1),y(k+1)));
end
```
plot(t,y)
title('your name, your student number, lesson number')
xlabel('time')
ylabel('speed')
The second numerical experiment is with the resistive force equal to

\[ 0.1 \text{abs}(u) + 0.001u^2. \]