Differential Equation Model.

Left Loop Equation:

\[ L \frac{d^2 Q}{dt^2} + R \frac{dQ}{dt} + \frac{1}{C} Q = V(t) = M \frac{di}{dt} \]

and \( Q(0) \) and \( Q'(0) \) are given,

and \( i \) is current in the right loop.

Charge in the left loop = \( Q \).

Current in the left loop is \( I = Q' \).
Center Loop Equation:

\[
(1/C) \ Q = x = \text{voltage drop in triode.}
\]

Right Loop Equation:

\[
i = a \ x - b \ x^3 \text{ where the a and b are constants,}
\]

\[
\text{which are to be determined via curve fitting.}
\]

Note,

\[
i' = a \ x' - b \ 3x^2x' \text{ and}
\]

\[
i' = a \ I/C - b \ 3(Q/C)^2 \ I/C.
\]
Upon scaling of time and the charge, we have

Van der Pol’s DE:

\[ y'' - \mu(1 - y^2)y' + y = 0. \]

Method of Solution.

The equivalent coupled system is

\[ y_1' = y_2 \quad \text{with } y_1(0) = y(0) \text{ and} \]

\[ y_2' = \mu(1 - y_1^2)y_2 - y_1 \quad \text{with } y_2(0) = y'(0). \]

Matlab's ode23s and ode15s can be used to solve this system, which is stiff for larger \( \mu \).
Matlab Implementation.

\begin{verbatim}
function ypvdpol=ypvdpol(t,y)

ypvdpol(1) = y(2);
ypvdpol(2) = 10*(1-y(1)^2)*y(2)-y(1);
ypvdpol= [ypvdpol(1) ypvdpol(2)]';

% your name , your student number, lesson number
clear;
tf = 100; % choose tf about ten times mu
yo = [2 0];
[t y] = ode23s('ypvdpol',[0 tf],yo);
plot(t,y(:,1));
title('your name, your student number, lesson number')
xlabel('time')
ylabel('charge')
%plot(y(:,1),y(:,2));
\end{verbatim}
Numerical Experiments.

In the first calculation we use $\mu = 10$ and the final time should be about ten times $\mu$.

The graph is for the scaled charge versus time.

The graph for the scaled current has a series of spikes.

In the Van der Pol circuit this might be series of flashes if the resistance was a light bulb.

This is a very stiff system.