Summary for MA 231 Test 1

Graphs of functions

Key ideas: Graphs of \( z = f(x, y) \). Level curves. Should be able to find the level curves of a function. Be able to recognize what types of level curves go with which graphs.

Taking Partial Derivatives

1. Need to be able to compute the first and second partials of a function \( f(x, y) \). \( \{f_x, f_y, f_{xx}, f_{xy}, f_{yy}\} \).

2. Be able to use both notations \( f_x \) and \( \frac{\partial f}{\partial x} \). Understand that \( f_x \) is the rate of change of \( f \) with respect to \( x \), also called the marginal \( f \) with respect to \( x \).

3. Word problems.

Chain rule

1. Be able to use the formula
\[
z' = f_x x' + f_y y'
\]
equivalently
\[
\frac{dz}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}
\]
to find derivatives (rate of change).

2. Be able to use the formula
\[
f(a + k, b + h) − f(a, b) \approx f_x(a, b)k + f_y(a, b)h
\]
or equivalently
\[
\Delta f = f(a + \Delta x, b + \Delta y) − f(a, b) \approx f_x(a, b)\Delta x + f_y(a, b)\Delta y
\]
(a) Given \( x', y', f_x, f_y, \) and \( f(a, b) \) estimate a different value \( f(a + k, b + h) \).
(b) Given \( x', y', f_x, f_y, \) and \( f(a, b) \) estimate the change \( \Delta f \).
Unconstrained Maximization/Minimization

1. Max or min a function \( f(x, y) \).
   (a) Use of first derivative test \( f_x = 0, f_y = 0 \)
   (b) Second derivative test with \( D = f_{xx}f_{yy} - (f_{xy})^2 \)
      i. \( D < 0 \): no max or min
      ii. \( D > 0 \): Is a max or min
         A. \( f_{xx} \) or \( f_{yy} > 0 \): Local min
         B. \( f_{xx} \) or \( f_{yy} < 0 \): Local max

2. Word Problems

Constrained optimization without multipliers

Max or Min \( f(x, y) \) given \( g(x, y) = 0 \). Or Max or Min \( f(x, y, z) \) given \( g(x, y, z) = 0 \).

1. Use constraint \( g = 0 \) to eliminate a variable from \( f \). Then solve the unconstrained problem that is left.

2. Word problems.

Least Squares

1. Given 3 or 4 data points \((x_i, y_i)\) be able to find \(A, B\) that give the best least squares fit of \(y = Ax + B\) to the data. Done by using calculus to minimize the sum of the square of the error 
   \(E_i = y_i - (Ax_i + B)\).

2. If given the general formula for \(A, B\) which uses \(\Sigma\) notation, and given some data, be able to use the formula to find \(A, B\).

Not on this test

Solving constrained optimization problems using Lagrange multipliers (Section 7.4) will be on the next test. It will not be on the Feb. 6 test.