CLT for Linear Spectral Statistics of Large Dimensional Sample Covariance Matrices

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Abstract

Let $B_n = (1/N)T_n^{1/2}X_nX_n^*T_n^{1/2}$ where $X_n = (X_{ij})$ is $n \times N$ with i.i.d. complex standardized entries having finite fourth moment, and $T_n^{1/2}$ is a Hermitian square root of the nonnegative definite Hermitian matrix $T_n$. The limiting behavior, as $n \to \infty$ with $n/N$ approaching a positive constant, of functionals of the eigenvalues of $B_n$, where each is given equal weight, is discussed. Due to the limiting behavior of the empirical spectral distribution of $B_n$, it is known that these linear spectral statistics (l.s.s.) converges a.s. to a non-random quantity. The talk outlines the latest results concerning their rate of convergence. It has been shown this rate to be $1/n$ by proving, after proper scaling, the l.s.s. form a tight sequence. Moreover, if $EX_{11}^2 = 0$ and $EX_{11}|X_{11}|^4 = 2$, or if $X_{11}$ and $T_n$ are real and $EX_{11}^4 = 3$, they have been shown to have Gaussian limits. (Joint work with Zhidong Bai.)