Chapter 1 Review Questions

1. Find all solutions to the linear system

\[ \begin{align*}
2x_1 + x_2 + 3x_3 + 2x_4 &= 5 \\
-x_2 + x_3 + 6x_4 &= 3
\end{align*} \]

2. Find all values of \( a \) for which the linear system has (a) no solution, (b) a unique solution, and (c) infinitely many solutions.

\[ \begin{align*}
x + 2y + 3z &= 4 \\
2x + y + 3z &= 5 \\
-3x - 3y + (a^2 - 5a)z &= a - 8.
\end{align*} \]

3. If possible, find the inverse of the following matrix:

\[ \begin{bmatrix}
1 & 2 & -1 \\
0 & 1 & 1 \\
1 & 0 & -1
\end{bmatrix}. \]

4. If \( A = \begin{bmatrix} -1 & -2 \\ -2 & 2 \end{bmatrix} \), find all values of \( \lambda \) for which the homogeneous system \( (\lambda I_2 - A)x = 0 \) has a nontrivial solution.

5. (a) If \( A^{-1} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & -1 & 4 \end{bmatrix} \) and \( B^{-1} = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 0 & -2 \\ 1 & 1 & -1 \end{bmatrix} \), compute \( (AB)^{-1} \).

(b) Solve \( Ax = b \) for \( x \) if \( A^{-1} = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 3 \\ 4 & 2 & 5 \end{bmatrix} \) and \( b = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \).

6. Find the \( LU \) factorization of the coefficient matrix of the linear system \( Ax = b \). Solve the system by using a forward substitution followed by a back substitution.

\[ A = \begin{bmatrix} 2 & 2 & -1 \\ -8 & -11 & 5 \\ 4 & 13 & -7 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ -14 \\ -5 \end{bmatrix}. \]

7. Answer each of the following as true or false. Explain your answers.

(a) If \( A \) and \( B \) are \( n \times n \) matrices, then \( (A + B)(A + B) = A^2 + 2AB + B^2 \).

(b) If \( \mathbf{u}_1 \) and \( \mathbf{u}_2 \) are solutions to the linear system \( A\mathbf{x} = \mathbf{b} \), then \( \mathbf{w} = \frac{1}{4}\mathbf{u}_1 + \frac{3}{4}\mathbf{u}_2 \) is also a solution to \( A\mathbf{x} = \mathbf{b} \).

(c) If \( A \) is a nonsingular matrix, then the homogeneous system \( A\mathbf{x} = \mathbf{0} \) has a nontrivial solution.

(d) A homogeneous system of three equations in four unknowns has a nontrivial solution.

(e) If \( A, B, \) and \( C \) are \( n \times n \) nonsingular matrices, then \( (ABC)^{-1} = C^{-1}B^{-1}A^{-1} \).