Instructions: If you can not do the required work in the space provided, use the back of these sheets (with a note saying which ones). Numbers in ( ) give the point allocation.

1. Perform one complete iteration of the simplex algorithm on:

(a) each of the following schemas:

\[ \begin{array}{c|cc}
 x_1 & x_2 & f(min) \\
 \hline
 2 & 2 & -1 \\
 3 & 2 & 0 \\
 2 & -1 & 1 \\
 \end{array} \]

\[ \begin{array}{c|cc}
 x_1 & x_2 & f(max) \\
 \hline
 4 & -2 & -1 \\
 0 & -1 & 1 \\
 1 & 0 & 2 \\
 \end{array} \]

Terminates on step 1 with optimal solution \( x^* = (0, 0, 0, 1) \)

(b) each of the following tableaus:

\[ \begin{array}{c|ccccc}
 x_1 & x_2 & x_3 & x_4 & x_5 & f(min) \\
 \hline
 4 & 2 & 0 & -1 & 0 & 0 \\
 3 & -2 & 0 & 3 & 0 & 1 \\
 1 & 0 & 0 & 2 & 1 & 0 \\
 1 & 1 & 1 & 0 & 0 & 0 \\
 \end{array} \]

Terminates on step 1 with \( f = -\infty \), \( x_1 \to +\infty \), \( x_2 = 0 \), \( x_3 \to +\infty \), \( x_4 \to +\infty \), \( x_5 = 0 \)

\[ \begin{array}{c|ccccc}
 x_1 & x_2 & x_3 & x_4 & x_5 & f(max) \\
 \hline
 2 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & 1 & 2 & -2 \\
 0 & 1 & 0 & 0 & 0 & 0 \\
 1 & 1 & 0 & 0 & 0 & 1 \\
 \end{array} \]

Terminates on step 2 with \( f = (5, 4, 1) \)
Transform:

(c) the second schema given in part (a) into an equivalent tableau:

\[
\begin{array}{ccc}
4 & -1 & 1 \\
0 & 1 & 0 \\
-1 & 1 & 1 \\
0 & 0 & 1 \\
\end{array}
\]

Which, if any, of the two

(e) schemas given in part (a) represent a degenerate basic feasible solution

(\textit{the second schema because } x_2 = 0) 

(f) tableaus given in part (b) have a constraint row that is not lexicographically positive

neither

2. Given that lexicographic ordering is being used to prevent cycling (20) while maximizing an objective function during an application of the simplex algorithm in uncontracted form.

(a) what property must the constraint rows of the input tableau have:

\textit{they must be lexicographically positive}

(b) what always happens to the distinguished element a

(i) during each pivot it does not increase

(ii) during each degenerate pivot it does not change

(iii) during each nondegenerate pivot it decreases

(c) what always happens to row 0 during each pivot

\textit{it decreases lexicographically}
3. Perform one complete iteration of phase I of the simplex method on (21) each of the following three schemas:

\[
\begin{array}{ccc}
 x_3 & x_1 & \text{-f(min)} \\
2 & 2 & -1 \\
3 & 1 & 0 \\
2 & 1 & 2
\end{array}
\quad
\begin{array}{ccc}
 x_3 & x_1 & \text{-f(max)} \\
2 & 2 & -1 \\
3 & 1 & 0 \\
2 & 1 & 2
\end{array}
\quad
\begin{array}{ccc}
 x_3 & x_2 & \text{-f(min)} \\
3 & 2 & -1 \\
-3 & 1 & 0 \\
2 & 1 & 2
\end{array}
\]

4. Given that phase II in contracted form (with schemas) terminates on (11) step 1 (with a basic optimal solution):

(a) what single property of the terminal schema would imply that there is only one optimal solution?

Distinguished row has no zeroes

(b) if the single property stated in part (a) is not present, does there have to be additional optimal solutions? Yes or No.

(Because pivoting in a column in which the distinguished row has a zero might be degenerate and hence not produce an additional optimal solution.)