Answer all parts. Your answers must be handed in to me in class on Wednesday, March 23.

1. Let the $2 \times 1$ vector $v_t$ be generated by the $VAR(1)$ process

$$v_t = \Theta v_{t-1} + w_t$$

where $\Theta$ has $i - j$th element $\theta_{ij}$ and $w_t \sim i.i.d.(0_{2 \times 1}, \Sigma)$.

(a) Show that $v_{i,t}$ has the following univariate representation

$$v_{i,t} = (\theta_{1,1} + \theta_{2,2})v_{i,t-1} - (\theta_{1,1}\theta_{2,2} - \theta_{1,2}\theta_{2,1})v_{i,t-2} + e_{i,t}$$

where

$$e_{1,t} = w_{1,t} - \theta_{2,2}w_{1,t-1} + \theta_{1,2}w_{2,t-1}$$
$$e_{2,t} = w_{2,t} - \theta_{1,1}w_{2,t-1} + \theta_{2,1}w_{1,t-1}$$

(b) Show that both $e_{1,t}$ and $e_{2,t}$ are covariance stationary and, in general, possess the auto-correlation function of a $MA(1)$ process.

(c) Given that $v_{i,t}$ has an $ARMA(p,q)$ representation, suggest values for $p$ and $q$.

(d) Show that the condition for stationarity of the $VAR(1)$ is equivalent to the stationarity conditions for the univariate models for $v_{1,t}$ and $v_{2,t}$.