

# Corrections to the first edition of the SIAM Frontier Book Series 33

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1. Page 8, change (1.12) to the following:

$$\beta^-(x) = \begin{cases} \beta(x), & \text{if } x \leq \alpha, \\ \beta(\alpha-) + \beta'(\alpha-)(x - \alpha)^1, & \text{if } x > \alpha, \end{cases}$$

$$\beta^+(x) = \begin{cases} \beta(\alpha+) + \beta'(\alpha+)(x - \alpha), & \text{if } x < \alpha, \\ \beta(x), & \text{if } x \geq \alpha. \end{cases}$$

Also add a sentence below (1.12): (If  $\beta'(\alpha\mp)$  does not exist, then we can use  $\beta^\mp(x) = \beta(\alpha\mp)$  for the extension).

2. Page 27, change (2.13) to the following:

$$C_j = \frac{v}{h^2}(x_{j+1} - \alpha) = v \delta_h(x_j - \alpha); \quad C_{j+1} = v \delta_h(x_{j+1} - \alpha),$$

That is, add  $v$  in the front of  $\delta_h(x_j - \alpha)$  and  $\delta_h(x_{j+1} - \alpha)$ .

3. Page 32, in caption of Table 3.2,  $\alpha = 1/3$  instead of  $\alpha = 1/2$ .
4. Page 35, one line below (3.11), change *third* to *fourth*.
5. Page 79. Change Lemma 5.2 and the proof to the following:

**Lemma 5.2** With the same notations and settings as in Theorem 5.1. Let  $(\xi, \eta)$  be the local coordinate system at a fixed point on the interface, and  $F(\mathbf{x}) = f(\mathbf{x}) - H(\varphi)\Delta\tilde{u}(\mathbf{x})$ . The following equality holds

$$\left[ \frac{\partial^2 q}{\partial \xi^2} \right] = [\Delta q] = [f - H(\varphi)\Delta\tilde{u}] = [F] \tag{5.25}$$

under the local coordinate system.

**Proof:** Since the Laplacian operator is invariant under orthogonal coordinates, we have

$$\Delta q = \frac{\partial^2 q}{\partial \xi^2} + \frac{\partial^2 q}{\partial \eta^2} = F.$$

From  $[q] = 0$ ,  $[q_\xi] = 0$ , and  $[q_\eta] = 0$  proved in Theorem 5.1, we can conclude that  $[\frac{\partial^2 q}{\partial \eta^2}] = 0$ , which is derived in §3.1 for the interface relations in the local coordinates. Therefore we conclude

$$[\Delta q] = [F] = \left[ \frac{\partial^2 q}{\partial \xi^2} \right]. \quad \square$$

6. Page 80-81, change all  $\frac{\partial^2 q}{\partial \mathbf{n}^2}(\mathbf{X}^*)$  to  $\frac{\partial^2 q}{\partial \xi^2}(\mathbf{X}^*)$ . It can be found in the following places:
  - The last line on page 80.
  - The second line on page 81.
  - Line 6 on page 81.
  - In (5.28) on page 81.
  - Two lines below (5.28) on page 81, (two places).
7. Page 81, In (5.28), the  $x$  in the partial derivative in the first term after '= $\Rightarrow$ ' should not be bold-faced, that is: change  $\frac{\partial^2 q}{\partial \mathbf{x}^2}(\mathbf{x}_{ij})$  to  $\frac{\partial^2 q}{\partial x^2}(\mathbf{x}_{ij})$
8. Page 94, line 10, the expression should be  $T\mathbf{G} + E\mathbf{U}(\mathbf{G}) + E\mathbf{U}(\mathbf{0})$ . The sign in the last term was wrong.
9. Page 170, last line, change (8.29) to the following:

$$\iint_{\Omega} \beta(x, y) \nabla u \cdot \nabla v \, dx dy = \iint_{\Omega} f v \, dx dy - \int_{\Gamma} v Q \, ds,$$

That is, the last term should have a negative sign.

10. Page 195,  $[u]_{;\tau}$  should  $[u_t]_{;\tau}$ .
11. Page 217, line 11:  $dx$  is missing in the right hand side of (10.8). The correct form should be:

$$\int_{\Omega} \boldsymbol{\psi} \cdot \nabla \chi \, dx = \int_{\Gamma} [\mathbf{n} \cdot \boldsymbol{\psi}] \chi \, ds - \int_{\Omega^+} \operatorname{div} \boldsymbol{\psi} \, \chi \, dx - \int_{\Omega^-} \operatorname{div} \boldsymbol{\psi} \, \chi \, dx,$$

12. Page 217, line 13, two lines below (10.8):  $dx$  should be  $ds$  in the first integral of the right hand side of the expression. The correct form should be:

$$-\int_{\Gamma}([\mathbf{n} \cdot \mathbf{S}] + \mathbf{f}) \cdot \boldsymbol{\phi} ds + \int_{\Omega^+} \operatorname{div} \mathbf{S} \cdot \boldsymbol{\phi} dx + \int_{\Omega^-} \operatorname{div} \mathbf{S} \cdot \boldsymbol{\phi} dx = \int_{\Omega} \mathbf{g} \cdot \boldsymbol{\phi} dx,$$

13. Page 217, the last line, (10.12):  $dx$  is missing in the middle integration. The correct form should be:

$$\int_{\Omega} \mathbf{S} : \nabla(\nabla\chi) dx + \int_{\Omega} \mathbf{g} \cdot \nabla\chi dx + \int_{\Gamma} \mathbf{f} \cdot \nabla\chi ds = 0,$$

14. Page 219, line 13-14: Change (10.22) to the following form:

$$\left[ \frac{\partial p}{\partial \mathbf{n}} \right] = [\mathbf{g} \cdot \mathbf{n}] + \frac{\partial}{\partial \boldsymbol{\tau}} \hat{f}_2 + 2 \left[ \mu \frac{\partial^2 \hat{\mathbf{u}}}{\partial \eta^2} \right] + 2\kappa \left( \left[ \mu \frac{\partial(\mathbf{u} \cdot \mathbf{n})}{\partial \mathbf{n}} \right] - \left[ \mu \frac{\partial(\mathbf{u} \cdot \boldsymbol{\tau})}{\partial \boldsymbol{\tau}} \right] \right)$$

15. Page 219, line 20: Add the following to the end of the paragraph:

Where  $(\xi, \eta)$  is the local coordinates at the particular point where the jumps are defined. We did not use the local coordinates for the first order derivatives since they are invariant under orthogonal coordinates transforms.

16. Page 220, line 6: The '+' sign in the first equality should be a '-' sign. The correct form should be:

$$[\mu u_{\mathbf{n}}] - [\mu v_{\boldsymbol{\tau}}] = 2[\mu u_{\mathbf{n}}] \quad \text{which is} \quad [\mu(u_x + v_y)] = 0.$$

17. Page 220, line 9: 'which is equivalent to (10.24)' should be 'which is equivalent to (10.23)'.

18. Page 220, last line but 9: 'from which we can get (10.24) ...', should be 'from which we can get (10.23) ...'.

19. Page 238, line 2-4: Change (10.60) to the following (only the last equality needs to be changed):

$$\begin{cases} \Delta p = \nabla \cdot \mathbf{g}, \\ [p] = \hat{f}_1 - 2 \frac{\partial \mathbf{q}}{\partial \boldsymbol{\tau}} \cdot \boldsymbol{\tau}, \\ \left[ \frac{\partial p}{\partial \mathbf{n}} \right] = \frac{\partial \hat{f}_2}{\partial \boldsymbol{\tau}} + 2 \frac{\partial^2}{\partial \eta^2} (\mathbf{q} \cdot \mathbf{n}) + 2\kappa \left( \left[ \frac{\partial(\tilde{\mathbf{u}} \cdot \mathbf{n})}{\partial \mathbf{n}} \right] - \left[ \frac{\partial(\mathbf{q} \cdot \boldsymbol{\tau})}{\partial \boldsymbol{\tau}} \right] \right) \end{cases}$$

20. Page 238, line 10, Add the following to the beginning of the line:

Where  $(\xi, \eta)$  is the local coordinates at the particular point where the jumps are defined. We did not use the local coordinates for the first order derivatives since they are invariant under orthogonal coordinates transforms.

21. Update the following references:

- [41]: Also in Advances in Computational Mathematics, in press.
- [120]: 19:1191-1197, 2006.
- [159]: Change the reference to: R. J. LeVeque. Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-Dependent Problems, SIAM Textbook Series, 2007.
- [163]: Change the reference to: "Z. Li, T. Lin, Y. Lin, R. C. Rogers, An immersed finite element space and its approximation capability, Numerical Methods for Partial Differential Equations, 20(3) 338-367,2004. DOI: 10.1002/num.10092".
- [176]: 1:874-885, 2006.
- [275]: 216:454-493, 2006.