Lab Session, Mathematics 587

Purpose: Learn Matlab *Partial Differential Equation* toolbox. Use it to solve some typical second order partial differential equations on some regions with certain boundary conditions; Visualize the mesh triangulation and the solutions; Export the triangulation.

Reference: Partial Differential Equation Toolbox, MathWorks.

<u>Test Problems</u>

1. Poisson equation on a unit circle:

$$\begin{aligned} -\Delta u &= 1, \qquad x^2+y^2<1 \\ u|_{\partial\Omega} &= 0, \qquad x \le 0 \\ u_n|_{\partial\Omega} &= 1, \qquad x > 0. \end{aligned}$$

2. Wave equation on a unit square $[-1 \quad 1] \times [-1 \quad 1]$:

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} &= \Delta u, \\ u(x,y,0) &= arctan\left(\cos\frac{\pi x}{2}\right) \\ u_t(x,y,0) &= 3\sin(\pi x)e^{\sin\frac{\pi y}{2}} \end{aligned}$$

$u = 0 \quad \text{on} \quad x=-1, \quad \text{and} \quad x=1, \qquad u_n = 0 \quad \text{on} \quad y=-1, \text{and} \quad y=1.$

3. Eigenvalue problem on a L-shape.

$$-\Delta u = \lambda u, \quad u = 0 \quad \text{on} \quad \partial\Omega.$$

The domain is the L-shape with corners (0,0), $(-1,0)$, $(-1,-1)$, $(1,-1)$, (1,1), and (0,1).

4. Heat equation: A heated metal block

$$\frac{\partial u}{\partial t} = \Delta u,$$

The domain is the rectangle $[-0.5 \quad 0.5] \times [-0.8 \quad 0.8]$ with a rectangle cavity $[-0.05 \quad 0.05] \times [-0.4 \quad 0.4]$. the boundary condition is the following:

- $u = 100$ on the left-hand side.
- $u = -10$ on the right-hand side.

- $u_n = 0$ on other boundaries.

5. Download the Matlab source code 2D.rar from

   `http://www4.ncsu.edu/~zhilin/TEACHING/MA587`

   Export the mesh of the last testing problem from Matlab and Run assemb.m to solve verify the example in the notes, page 100-101.

General Procedure

- Draw the geometry.
- Define the boundary conditions.
- Define the partial differential equation.
- Define the initial conditions if necessary.
- Solve the PDE.
- Plot the solution.
- Refine the mesh if necessary.
- Save and quit.