

Midterm Review

1. Midterm: in class, closed to books and notes, and other materials.
2. Homework problems. Problems asked in class.
3. Finite difference procedure, grid, “derivative \implies finite difference”, algebraic system (linear or nonlinear system of equations), error analysis, validation.
4. How to derive finite difference schemes? Basic finite difference formula for u_x and u_{xx} , finite difference operator and symbolic derivation, un-determined coefficient method. How to derive finite difference formulas for u_{xxx} , u_{ttxx} ?
5. Finite difference schemes for elliptic equations. How to discretize a self-adjoint differential operator $\nabla \cdot (p\nabla u)$?
6. How to deal with difference boundary conditions? Dirichlet, Neumann, mixed, and periodic. How do they affect the matrix-vector form? How do you apply the ghost point method?
7. Error analysis. What is the definition of the local truncation error, consistency, stability, and convergence? What conditions can guarantee the convergence? From convergence, can we conclude the consistency and stability? Pay particular attention to the definition of the consistency for time dependent problems.
8. von-Neumann stability analysis in 1D and 2D using the simplified form.
9. Derive different finite difference methods for various problems both in component form and matrix-vector form. For elliptic problem and implicit methods, also know how to use the Jacobi, Gauss-Seidel, and $\text{SOR}(\omega)$ iterative methods.
10. How do you validate a numerical methods? What is the grid refinement analysis?
11. Know the advantages and limitations of finite difference methods compared with other method, for example, finite element methods, and different difference methods for a same problem. For example, compare among FW-CT, BW-CT, Crank-Nicolson, ADI methods for heat equations.
12. What is the method of line (MOL) for 2D heat equations?
13. What is the maximum principle (continuous and discrete) for elliptic problems? How is it used to show the second order convergence for finite difference method?