Selected solutions

1 You need to use the condition $\mathbf{x}^* = R\mathbf{x}^* + c$ to get $\mathbf{x}^{k+1} - \mathbf{x}^k = R\left(\mathbf{x}^k - \mathbf{x}^*\right) - \left(\mathbf{x}^k - \mathbf{x}^*\right) = R^k(\mathbf{x}^1 - \mathbf{x}^0)$.

2 (a) The intervals are: $-2 \le \lambda_3 \le 2$; $-7 \le \lambda_2 \le -5$; $8 \le \lambda_1 \le 10$;

(d) If we take $\mathbf{x}_0 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$, then we have $\mathbf{x}_1 = \begin{bmatrix} \frac{2}{129}\sqrt{129} & -\frac{5}{129}\sqrt{129} & \frac{10}{129}\sqrt{129} \end{bmatrix}^T$; $\mu_1 = 5.4264$.

If we use the x_p, i.e. the infinity norm scaling, we would have i$y_1 = Ax_0$, $\mathbf{x}^1 = y_1/(y_1)_p = \begin{bmatrix} 1/5 & -1/2 & 1 \end{bmatrix}^T$; $\mu_1 = (y_1)_p = 10$.

(e) We have $\mathbf{x}^1 = \begin{bmatrix} 0.1534 & -0.9860 & 0.0657 \end{bmatrix}^T$; $\mu_1 = -6.1512$.

3 For the first matrix, all the eigenvalues are distinct, so the essential condition $|\lambda_n| < |\lambda_{n-1}|$, so the inverse Power method converges quadratically since $A = A^T$.

For the second matrix, when n is even, A is called skew-symmetric. It is easy to show that all the eigenvalues are complex numbers in pair. Thus the essential condition $|\lambda_n| < |\lambda_{n-1}|$ is violated. The method will not give the least dominant eigenvalue. When n is odd, then one can show that least dominant eigenvalue is $\lambda_n = 2$ and it is simple eigenvalue. Thus the inverse Power method converges even though it may be slow.

4 (a), No since $\|x\|_2 \ne \|y\|_2$. (b) $\alpha = \sqrt{14}$ since $y = -e_4$.

5 (a) $\|r(x)\|_2 = \sqrt{\|b_1\|_2 + \|b_2 - Bx\|_2 + \|b_3\|_2}$. Thus the minimum is $\|r(x^*)\|_2 = \sqrt{\|b_1\|_2 + \|b_3\|_2}$ when $Bx = b_2$ or $x = B^{-1}b_2$.

(b): $x^* = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$ and $\|r(x^*)\|_2 = \sqrt{3}$.

For the second problem, we have $x^* = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ and $\|r(x^*)\|_2 = \sqrt{65}$.

6 The solution is $x^* = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ and $\|r(x^*)\|_2 = 1$.

7 The solution is $U = I_2$, $\Sigma = \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$; and $V = \begin{bmatrix} 1/2\sqrt{2} & 0 & -1/2\sqrt{2} \\ 0 & 1 & 0 \\ 1/2\sqrt{2} & 0 & 1/2\sqrt{2} \end{bmatrix}$.