MA402, Homework #1
Due Tuesday, September 9, 2001

1. Mathematically, we know that

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots \tag{1}$$

We can use the truncated Taylor series

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} \tag{2}$$

to approximate e^x. Write a matlab script file to evaluate e^x and find out the absolute and relative errors of your computed result(s) at x for $n = 5, 10, 20$. **Hint:** Complete the Matlab code below

```
x=1; n=5;
my_ex = 1; my_fac = 1; my_xn = 1;
for i=1:n
   my_fac = my_fac*i;                        % Get i!
   my_xn = my_xn* ?;                         % Get x^n
   my_ex = ? + my_xn/?;
end
my_abse = abs( exp(x) - ?)                  % Get the absolute error
my_rele = abs( exp(x) - ?)/?                % Get the relative error
```

and test it for $x = -1$, $x = 0$, and $x = 1$. Record and arrange your results.

Extra Credit: 5 points. Convert your code either in **C** or **Fortran**.

2. Rewrite the following expressions to avoid any possible loss of accuracy.
   (a) $\dfrac{e^x - 1 - x}{x^2}$ for x near 0.
   (b) $\sin(3x) - \sin(x)$ for x near 0.
   (c) $\sqrt{x^2+1} - x$ for very large (but within the computer number system) x.

3. Consider the explicit finite difference model for the heat transfer in a thin rod, *which may have heat loss on the lateral surface*. See Chapter 1.2 and 1.3 for details.
   (a) Modify the code *heat.m* to implement the discrete model.
   (b) Duplicate some of the calculations in Chapter 1.2 and 1.3. Find an approximate stability condition. How is it different from the text?
   (c) Experiment with different values of c (the lateral "insulation" coefficient.) What happens to the numerical solution as c increases/decreases?

   The number of plots should be between three to six. You need to label all your plots.