

22-May		23-May			24-May			25-May			
Zhilin Li(Chair )		Gang Bao(Chair )			Shi Jin (Chair )			P. Meniv (Chair )			
8:30-8:50	Yongzhong Song	8:30-9:05		P. Zhang		8:30-9:05		Tao Tang		8:30-9:05	Y-Q Huang
	Zhongci Shi	9:05-9:40		A. Fogelson		9:05-9:40		Chaouqi Misbah		9:05-9:40	BC-Khoo
8:50-9:30	Weinan E	9:40-10:15		J. Shen		9:40-10:15		W. Cai		9:40-10:15	K. Ito
<b>9:30-10:00</b>	<b>Photo &amp; Coffee</b>										
<b>Qiang Du (Chair )</b>		<b>10:15-10:35</b>		<b>Coffee</b>		<b>10:15-10:35</b>		<b>Coffee</b>		<b>10:15-10:35</b>	<b>Coffee</b>
10:00-10:35	P. Angot	<b>X-P. Wang (Chair )</b>			<b>Y-Q. Huang (Chair )</b>			<b>A. Chertock (Chair )</b>			
10:35-11:10	X-P Wang	10:35-11:10		P. Meniv		10:35-11:10		Zhimin Chen		10:35-11:10	S. Jin
11:10-11:45	Guowei He	11:10-11:45		I. Klapper		11:10-11:45		A. Chertock		11:10-11:45	Weimin Han
11:45-12:20	Z. Zhang	11:45-12:20		H. Lee		11:45-12:20		P. Lin		11:45-12:20	Qi Wang
<b>12:30-13:10</b>	<b>Lunch</b>	<b>12:30-13:10</b>		<b>Lunch</b>		<b>12:30-13:10</b>		<b>Lunch</b>		<b>12:30-13:10</b>	<b>Lunch</b>
<b>P. Zhang (Chair )</b>		<b>Z-M Chen (Chair )</b>			<b>Tao Tang (Chair )</b>		<b>W. Cai (Chair )</b>		<b>14:00 Tour</b>		
14:00-14:35	Q. Du	14:00-14:35		G. Bao		14:00-14:35	Bin Wang	14:00-14:35			T-G. Liu
14:35-15:10	B. Dillon	14:35-15:10		B. Guy		14:35-15:10	L. Zhu	14:35-15:10			J-F. Yin
15:10-15:45	H. Liu	15:10-15:45		J-G Huang		15:10-15:45	A. Kurganov	15:10-15:45			Z-J Tan
15:45-16:20	Wenbing Chen	15:45-16:20		J. Hu		15:45-16:20	H-J Wu	15:45-16:20			Cheng Wang
<b>16:20-16:40</b>	<b>Coffee</b>	<b>16:20-16:40</b>		<b>Coffee</b>		<b>16:20-16:40</b>		<b>Coffee</b>			
<b>J. Shen (Chair )</b>		<b>Z. Zhang (Chair )</b>		<b>Sheng Xu (Chair )</b>		<b>P. Lin (Chair )</b>		<b>T. Lin (Chair )</b>			
16:40-17:10	Tao Lin	16:40-17:10	L-L. Ju	16:40-17:10	Ruiz Alvarez Juan	16:40-17:10	Guang Lin	16:40-16:55			Wenyuan Liao
17:10-17:40	M. Burkow	17:10-17:40	H. Wang	17:10-17:40	J-J. Xu	17:10-17:40	S. Xu	16:55-17:10			Yu Liu
17:40-18:10	Yuan Li	17:40-18:10	Xiaoqin Jin	17:40-17:55	Bo Wang	17:40-18:10	LL Wang	17:10-17:25			Abouzar Kaboudian
18:10-18:40	Z-H Qiao	18:10-18:40	S. Zhao	17:55-18:10	Yanqiu Wang	18:10-18:40	Xiu Ye	17:25-17:40	Zhenlin Guo		
				18:10-18:25	Qinhai Zhang			17:40-17:55	Shumo Cui		
				18:25-18:40	Runchang Lin			17:55-18:10	Gang Li		
								18:10-18:25	Q-X. Wang		
<b>18:50-19:30</b>	<b>Dinner</b>	<b>19:00</b>		<b>Banquet</b>		<b>18:50-19:30</b>		<b>Dinner</b>		<b>18:30-19:30</b>	<b>Dinner</b>

# PROGRAM

**May 21: Registration and arrival**

---

**Place: Dinner Hall, 2nd Floor, NanShan Hotel (East Building)**  
**Dinner, 18:30-19:30**

**May 22**

---

**Opening Ceremony Chair: Zhilin Li (North Carolina State University )**  
**Place: Conference Room in 3rd floor, NanShan Hotel**

- 8:30-8:50, Opening remarks: Yongzhong Song (NNU), Zhongci Shi (CAS)

**Morning First Session**

**Place: Conference Room in 3rd floor, NanShan Hotel**

- 8:50-9:30, Weinan E (Princeton University)

*Modeling rare events*

**9:30-10:00, Conference photograph**  
**Coffee break**

**Morning Second Session Chair: Qiang Du (Penn State, USA)**

**Place: Conference Room in 3rd floor, NanShan Hotel**

- 10:00-10:35, Philippe Angot (Aix-Marseille Universit, France)

*Recent results with the fast vector penalty-projection methods for incompressible multi-phase viscous flows*

- 10:35-11:10, Xiaoping Wang (Hong Kong University of Science and Technology)

*Analysis and simulation of wetting and contact angle hysteresis on rough surfaces*

- 11:10-11:45, Guowei He (LNM, Institute of Mechanics, Chinese Academy of Science)

*A hybrid simulation of large-eddy simulation and immersed boundary method for bio-locomotion at moderate Reynolds numbers*

- 11:45-12:20, Zhiming Zhang (Washington State University, USA)

*Super-convergence: Unclaimed Territories*

---

**Lunch 12:30**

---

**Afternoon First Session Chair: Pingwen Zhang (Peking University)**

**14:00-16:20**

**Place: Main Conference Room in 3rd floor, NanShan Hotel**

- 14:00-14:35, Qiang Du (Penn State University, USA)  
*How to compute saddle point*
- 14:35-15:10, Robert Dillon (Washington State University, USA)  
*Models for complex fluids-structure interaction in peristalsis and sperm motility*
- 15:10-15:45, Hailiang Liu (Iowa State University, USA)  
*Entropy satisfying methods for kinetic Fokker-Planck equations*
- 15:45-16:20, Wenbin Chen (Fudan University)  
*Energy-conserved splitting finite-difference time-domain methods for Maxwell's equations*

**16:20-16:40, Coffee break**

**Afternoon Second Session Chair: Jie Shen (Purdue University & Xiamen University)**

16:40-18:40

**Place: Conference Room in 3rd floor, NanShan Hotel**

- 16:40-17:10, Tao Lin (Virginia Tech., USA)  
*Immersed finite element methods for planar elasticity interface problems*
- 17:10-17:40, Burkow Markus (University of Bonn Wegelerstrasse)  
*Numerical simulation of current driven sediment transport processes and resulting*
- 17:40-18:10, Yuan Li (CAS, China)  
*HLLC-based flow solver for the seven-equation compressible two-phase model and its implementation on GPU*
- 18:10-18:40, Zhonghua Qiao (Hong Kong Polytechnic University)  
*Stability and convergence analysis of the nonlinear epitaxial growth model without slope selection*

**Dinner, 18:50-19:30**

**May 23**

---

**Morning First Session Chair: Gang Bao (Michigan State Univ. USA & Zhejiang Univ. China)**

**Place: Conference Room in 3rd floor, NanShan Hotel**

- 8:30-9:05, Pingwen Zhang (Peking University, China)  
*Numerical methods of quasi-crystals*
- 9:05-9:40, Aaron Fogelson (University of Utah, USA)  
*Interface problems for two-fluid mixtures*

- 9:40-10:15, Jie Shen (Purdue University)

*Some recent advances on phase-field models for multiphase complex fluids*

**10:15-10:35, Coffee break**

**Morning Second Session Chair: Xiao-Ping Wang ( Hong Kong University of Science and Technology)**

**Place: Conference Room in 3rd floor, NanShan Hotel**

- 10:35-11:10, P. Meniv (University of Alberta)

*A direction splitting algorithm for flow problems in complex/moving geometries*

- 11:10-11:45, Isaac Klapper (Montana State University, USA)

*Microbial-induced mineralization in biofilms*

- 11:45-12:20, Hyung-Chun Lee (Ajou University, Korea)

*A stochastic sparse grid collocation method for stochastic optimal control problems*

---

**Lunch 12:30**

---

**Afternoon First Session Chair: Zhiming Chen (CAS, China)**

14:00-16:20

**Place: Main Conference Room in 3rd floor, NanShan Hotel**

- 14:00-14:35, Gang Bao (Michigan State Univ., USA and Zhejiang Univ., China)

*Mathematical and computational challenges for inverse scattering in wave propagation*

- 14:35-15:10, Robert Guy (University of California at Davis, USA)

*A multigrid method for solving implicit-time immersed boundary equations*

- 15:10-15:45, Jinguo Huang (Shanghai Jiao Tong University, China)

*The compact discontinuous Galerkin method for nearly incompressible linear elasticity*

- 15:45-16:20, Jun Hu (Peking University)

*The minimal finite element method for the linear elasticity problem within the stress-displacement formulation*

**16:20-16:40, Coffee break**

**Afternoon Second Session Chair: Zhimin Zhang (Wayne State University, USA)**

16:40-18:40

**Place: Conference Room in 3rd floor, NanShan Hotel**

- 16:40-17:10, Lili Ju (University of South Carolina)

*A posteriori error analysis of finite element methods for linear nonlocal diffusion and peridynamic models*

- 17:10-17:40, Hong Wang (University of South Carolina)  
*A fast finite difference method for three-dimensional space-fractional diffusion equations*
- 17:40-18:10, Xiaoqing Jin (University of Macau)  
*A sinc collocation method for Love's integral equation with a very small parameter*
- 18:10-18:40, Shan Zhao (University of Alabama)  
*Time domain interface methods for electromagnetic wave propagation in dispersive media*

**Afternoon Third Session Chair: Sheng Xu (SMU, USA)**

16:40-18:40

**Place: Conference Room in first floor, NanShan Hotel**

- 16:40-17:10, Juan Ruiz álvarez (Universidad de Alcalá de Henares)  
*The immersed interface method for axis-symmetrical geometries*
- 17:10-17:40, Jianjun Xu (Xiangtan University)  
*A coupled immersed interface and level set method for three-dimensional interfacial flows with insoluble surfactant*
- 17:40-17:55, Bo Wang (National University of Singapore)  
*Hybridizable discontinuous Galerkin method (HDG) for Stokes interface flow*
- 17:55-18:10, Yanqiu Wang (Oklahoma State University, USA)  
*Weak Galerkin method for the Ciarlet-Raviart mixed formulation of biharmonic equations*
- 18:10-18:25, Qinghai Zhang (University of California at Davis)  
*Resolving the boundary-layer on your laptop: A fourth-order projection method*
- 18:25-18:40, Runchang Lin, Texas A&M International University, USA  
*A balanced finite element method for singularly perturbed reaction-diffusion problems*

**Banquet 19:00**

**May 24**

---

**Morning First Session Chair: Shi Jin(University of Wisconsin-Madison )**

**Place: Meeting Room in 3rd floor, NanShan Hotel**

- 8:30-9:05, Tao Tang (Hong Kong Baptist University)  
*Spectral deferred correction methods on general quadrature nodes*
- 9:05-9:40, Chaouqi Misbah (CNRS, Grenoble, France)  
*Towards a bottom-up approach to blood flow*

- 9:40-10:15, Wei Cai (University of North Carolina at Charlotte, USA)  
*Monte Carlo walk on spheres method for charge density on a surface at non-constant potentials*

**10:15-10:35, Coffee Break**

**Morning Second Session Chair: Yunqing Huang (Xiangtan University )**

**Place: Meeting Room in 3rd floor, NanShan Hotel**

- 10:35-11:10, Zhiming Chen (Chinese Academy of Sciences)  
*An adaptive immersed finite element method with arbitrary Lagrangian-Eulerian scheme for Parabolic equations in variable domains*
- 11:10-11:45, Alina Chertock (North Carolina State University, USA)  
*Central-upwind scheme for the shallow water system with Thermodynamics*
- 11:45-12:20, Ping Lin (University of Dundee, UK)  
*Phase field model of Thermo-Induced Marangoni effects and instability in a two-fluid mixture using a finite element method*

---

**Lunch 12:30**

**Afternoon First Session Chair: Tao Tang(Hong Kong Baptist University )**

14:00-16:20

**Place: Meeting Room in the 3rd floor, NanShan Hotel**

- 14:00-14:35, Bin Wang (Qinhua University)  
*Introduction to decadal predictions by FGOALS-g2*
- 14:35-15:10, Luoding Zhu (Indiana University - Purdue University)  
*A 3D immersed boundary method with application*
- 15:10-15:45, Alexander Kurganov (Tulane University)  
*New adaptive artificial viscosity method for hyperbolic systems of conservation laws*
- 15:45-16:20, Haijun Wu (Nanjing University)  
*An unfitted HP-interface penalty finite element method for elliptic interface problems*

**Afternoon Second Session: Chair: Wei Cai (UNCC, USA)**

14:00-16:20

**Place: Meeting Room in the first floor, NanShan Hotel**

- 14:00-14:35, Tiegang Liu (Beihang University)  
*Hybrid RKDG and WENO methods for hyperbolic conservation laws*
- 14:35-15:10, Junfeng Yin (Tongji University)  
*Modulus-based successive over-relaxation method for pricing American options*

- 15:10-15:45, Zhijun Tan (Sun Yat-sen University)  
*Solving two-phase flows with interfaces and irregular domains by an IIM*
- 15:45-16:20, Cheng Wang ( University of Massachusetts at North Dartmouth, USA)  
*Global in time numerical stability for time dependent nonlinear PDEs*

**16:20-16:40 Coffee break**

**Afternoon Third Session: Chair: Ping Lin (University of Dundee, UK)**

16:40-18:40

**Place: Meeting Room in 3rd floor, NanShan Hotel**

- 16:40-17:10, Guang Lin (Pacific Northwest National Laboratory, USA)  
*Uncertainty quantification, modeling and simulation for physical and biological flow problems*
- 17:10-17:40: Sheng Xu, Southern Methodist University, USA  
*The immersed interface method for two-fluid flows*
- 17:40-18:10, Lilian Wang (Nanyang Technological University, Singapore)  
*Fast time-domain computation of wave scattering problems*
- 18:10-18:40: Xiu Ye (University of Arkansas at Little Rock)  
*Weak Galerkin finite element methods for partial differential equations*

**Afternoon Fourth Session: Chair: Tao Lin (Virginia Tech, USA)**

16:40-18:25

**Place: Meeting Room in the first floor, NanShan Hotel**

- 16:40-16:55: Wenyan Liao (Univeristy of Calgary)  
*On the stability of a class of Rosenbrock methods and applications in solving parabolic problems*
- 16:55-17:10, Yu Liu (Tulane University)  
*Central-upwind scheme for the shallow water system with thermodynamics*
- 17:10-17:25: Abouzar Kaboudian (National University of Singapore)  
*The ghost solid method for the elastic solid-solid interface*
- 17:25-17:40, Zhenlin Guo (University of Science and Technology Beijing, China)  
*Phase field model of Thermo-Induced Marangoni effects and Marangoni-Rayleigh-Benard instability in a two-fluid mixture using a finite element method*
- 17:40-17:55, Shumo Cui(Tulane University)  
*Application of deterministic particle methods to PDEs arising in financial modeling*

- 17:55-18:10, Gang Li (Qingdao University, China)  
*Hybrid well-balanced WENO schemes with different indicators for shallow water equations*
- 18:10-18:25, Quanxiang Wang (Nanjing Normal University)  
*Fourier finite volume element method for solving global quasi-geostrophic equations*

**Dinner, 18:50-19:30**

**May 25, Half day conference**

---

**Morning First Session Chair: P. Meniv (University of Alberta, Canada)**  
**Place: Meeting Room in the 3rd floor, NanShan Hotel**

- 8:30-9:05, Yunqing Huang (Xiangtan University)  
*Finite element analysis of electromagnetics in metamaterials*
- 9:05-9:40, Khoo B.C. (National University of Singapore)  
*Dynamics of supercavitating flow: Use of a Homogenous flow method*
- 9:40-10:15, Kazufumi Ito (North Carolina State University, USA)  
*Nonsmooth Optimization, Theory and Applications*

**10:15-10:40, Coffee break**

**Morning Second Session Chair: A. Chertock (NCSU, USA)**  
**Place: Meeting Room in the 3rd floor, NanShan Hotel**

- 10:40-11:15, Shi Jin (University of Wisconsin-Madison USA & Shanghai Jiatong Univ.)  
*Asymptotic-Preserving schemes for kinetic-fluid modeling of two phase flows*
- 11:15-11:50, Weimin Han (University of Iowa, USA)  
*A family of models in X-ray dark-field tomography*
- 11:50-12:25, Qi Wang (University of South Carolina, USA)  
*Active material systems and pattern formation*

---

**Lunch 12:30**

---

**Afternoon: 14:00-18:00, Half-day guided city tour**

**May 26**

- 
- Departure



# ABSTRACT

## Recent results with the fast vector penalty-projection methods for incompressible multiphase viscous flows

Philippe Angot

*Aix-Marseille Université & LATP – CMI, CNRS UMR 7353  
39 rue F. Joliot Curie, 13453 Marseille Cedex 13, France.*

The family of so-called *vector penalty-projection* methods has been designed to overcome the major drawbacks of the usual projection splitting methods or augmented Lagrangian iterative methods to solve unsteady incompressible multiphase Navier-Stokes problems; see [1]. The new fast version ( $VPP_\varepsilon$ ) of these methods [2, 4] proves to be really efficient and impressive to solve also anisotropic Darcy problems or fluid-structure interaction problems within an Eulerian formulation and immersed boundaries.

We propose first to explain the key ideas governing the construction of such methods giving a divergence as  $\mathcal{O}(\varepsilon \delta t)$ ,  $\varepsilon$  being the penalty parameter chosen as small as desired until machine precision and  $\delta t$  being the time step. Besides, the original vector penalty-correction step in the splitting algorithm has a very good conditioning quasi-independent of the spatial mesh step.

Second, we review some recent results, both theoretical [3, 7, 5] or numerical [4, 5, 6]. They show that the ( $VPP_\varepsilon$ ) method is very robust whatever the density, viscosity or anisotropic permeability jumps. Moreover, this method is running faster than usual methods and being able to efficiently and accurately compute sharp test cases whereas other methods crash.

## References

- [1] PH. ANGOT, J.-P. CALTAGIRONE AND P. FABRIE, Vector penalty-projection methods for the solution of unsteady incompressible flows, in *Finite Volumes for Complex Applications V - Problems & Perspectives*, R. Eymard and J.-M. Hérard (Eds), pp. 169–176, ISTE Ltd and J. Wiley & Sons, 2008.
- [2] PH. ANGOT, J.-P. CALTAGIRONE AND P. FABRIE, A spectacular vector penalty-projection method for Darcy and Navier-Stokes problems, in *Finite Volumes for Complex Applications VI - Problems & Perspectives*, J. Fořt et al. (Eds), Springer Proceedings in Mathematics **4**, Vol. 1, pp. 39–47, Springer-Verlag (Berlin), 2011.
- [3] PH. ANGOT, J.-P. CALTAGIRONE AND P. FABRIE, A new fast method to compute saddle-points in constrained optimization and applications, *Applied Mathematics Letters* **25**(3), 245–251, 2012.

- [4] PH. ANGOT, J.-P. CALTAGIRONE AND P. FABRIE, A fast vector penalty-projection method for incompressible non-homogeneous or multiphase Navier-Stokes problems, *Applied Mathematics Letters* **25**, 2012 (in press).
- [5] PH. ANGOT, J.-P. CALTAGIRONE AND P. FABRIE, A fast vector penalty-projection method for incompressible Darcy and multiphase Navier-Stokes problems, *Numerische Mathematik*, preprint 2011 (submitted).
- [6] PH. ANGOT AND R. CHEAYTOU, Vector penalty-projection method for incompressible fluid flows with open boundary conditions, *J. Comput. Phys.*, preprint 2012 (submitted).
- [7] PH. ANGOT AND P. FABRIE, Convergence results for the vector penalty-projection and two-step artificial compressibility methods, *Discrete and Continuous Dynamical Systems, Series B*, **17**(5), 1383–1405, 2012.

## **Mathematical and computational challenges for inverse scattering in wave propagation**

**Gang Bao**

*Zhejiang University and Michigan State University*

Inverse scattering problems are classical problems in mathematical physics, which aim to determine physical properties or structures of the medium and materials from additional measured data generated by an incident wave field. Because of significant applications in diverse areas of science and engineering, inverse scattering problems have made continuous progress. There are emerging new problems and challenges both in terms of applications and method developments. The speaker will present the general model problems, discuss recent developments, and highlight some ongoing and future directions.

## **Monte Carlo walk on spheres method for charge density on a surface at non-constant potentials**

**Wei Cai**

*University of North Carolina at Charlotte & Shanghai Jiaotong University*

Monte Carlo walk on spheres (WOS) is a well known method to solve potential problem of elliptic partial differential equation from electrostatic. And, the charge density on a conducting surface, related to the normal derivative of the potential, can be found by a last-passage method using WOS, which calculates the charge density at a single point. In this paper, we will apply the WOS for computing the charge density on surfaces with non-constant potentials. First, the charge density is represented as a hypersingular boundary integral equation (BIE) using the potential on a small hemisphere centered at the location of the sought-after charge density, then, the potential at Gauss quadrature points on the hemisphere is obtained by the WOS method. The resulting BIE-WOS will be a hybrid method combining the deterministic BIE method and the random WOS method, resulting in an efficient local method for charge density on any variable potential surface where solution of the associated Laplace equation is only calculated around the location of the needed charge density. Numerical tests demonstrate the efficiency and accuracy of the proposed method. (joint work with Changhao Yan and Xuan Zeng, Fudan University)

## **Energy-conserved splitting finite-difference time-domain methods for Maxwell's equations**

**Wenbin Chen**

*Fudan University*

In this talk, we develop and analyze efficient energy-conserved splitting finite-difference time-domain (FDTD) schemes for solving Maxwell's equations in electromagnetic computations. All proposed energy-conserved splitting finite-difference time-domain (EC-S-FDTD) algorithms are strictly proved to be energy-conserved and unconditionally stable, and they can be computed efficiently. Rigorous convergence results are obtained for the schemes. The EC-S-FDTDII schemes are proved to have second order in both time step and spatial steps, while the EC-S-FDTDI schemes have second order in spatial steps and first order in time step. The error estimates are optimal, and especially the constant in the error estimates is proved to be only  $O(\tau)$ . Numerical experiments confirm the theoretical analysis results.

## **An adaptive immersed finite element method with arbitrary Lagrangian-Eulerian scheme for Parabolic equations in variable domains**

**Zhiming Chen**

*Institute of Computational Mathematics, Chinese Academy of Sciences*

An adaptive immersed finite element method based on the a posteriori error estimate for solving elliptic equations with non-homogeneous boundary condition in general Lipschitz domain is proposed. The underlying finite element mesh need not to fit the boundary of the domain. Optimal a priori error estimate of the proposed immersed finite element method is proved. The immersed finite element method is then used to solve parabolic problems in time variable domains together with an arbitrary Lagrangian-Eulerian (ALE) time discretization scheme. An a posteriori error estimate for the fully discrete immersed finite element method is derived which can be used to adaptively update the time step sizes and finite element meshes at each time step. Numerical results are reported to support the theoretical results. This is a joint work with Zedong Wu and Yuanming Xiao.

## **Central-upwind scheme for the shallow water system with Thermodynamics**

**Alina Chertock<sup>a</sup>**

**Alexander Kurganov<sup>b</sup>**

**Yu Liu<sup>c</sup>**

<sup>a</sup>*Department of Mathematics, North Carolina State University, Raleigh, NC, 27695, USA;*

<sup>b</sup>*Mathematics Department, Tulane University, New Orleans, LA 70118, USA;*

<sup>c</sup>*Mathematics Department, Tulane University, New Orleans, LA 70118, USA.*

We introduce a central-upwind scheme for one- and two-dimensional systems of shallow-water equations with horizontal temperature gradients (the Ripa system). The scheme is well-balanced, positivity preserving and does not develop spurious pressure oscillations in the neighborhood of temperature jumps, that is, near the contact waves. Such oscillations would typically appear when a conventional Godunov-type finite volume method is applied to the Ripa system, and the nature of the oscillation is similar to the ones appearing at material interfaces in compressible multifluid computations. The idea behind the proposed approach is to utilize the interface tracking method, originally developed in [A. CHERTOCK, S. KARNI, A. KURGANOV, *M2AN Math. Model. Numer. Anal.*, 42(2008), pp. 991-1019] for compressible multifluids. The resulting scheme is highly accurate, preserves two types of “lake at rest” steady states, and is oscillation free across the temperature jumps, as it is illustrated in a number of numerical experiments.

# Application of deterministic particle methods to PDEs arising in financial modeling

Shumo Cui

*Mathematics Department, Tulane University, New Orleans, LA 70118, USA.*

We numerically study convection-diffusion equations arising in financial modeling. We focus on the convection-dominated case, in which the diffusion coefficients are small. Both finite-difference and Monte-Carlo methods which are widely used in the problems of this kind are typically inefficient due to severe restrictions on the meshsize and the number of realizations needed to achieve high resolution.

We develop new deterministic particle methods which introduce very small numerical diffusion and do not suffer from the aforementioned drawbacks. Our approach is based on the operator splitting: The hyperbolic steps are made using the method of characteristics, while the parabolic steps are performed using a special discretization of the integral representation of the solution.

We apply our scheme to a variety of test problems and the numerical results clearly demonstrate a high accuracy, efficiency and robustness of the proposed method.

This is a joint work with Alexander Kurganov (Tulane University) and Alexei Medovikov (Susquehanna International Group).

# Models for complex fluids-structure interaction in peristalsis and sperm motility

Robert Dillon

*Washington State University*

While much progress has been made in the development of mathematical models and numerical methods for fluid-structure interactions in a Newtonian fluid, much work needs to be done in the case of complex fluids. We describe a Lagrangian mesh method for modeling complex fluids where the fluid viscoelasticity is represented by a discrete network of Maxwell elements. The rheological properties of the Lagrangian mesh fluid is compared with an Oldroyd-B model for complex fluids in a computational rheometer. We show simulation results from immersed boundary models for peristalsis and sperm motility in Lagrangian mesh and Oldroyd-B fluids.

## How to compute saddle point

**Qiang Du**

*Penn State University*

Exploring complex energy landscape is a challenging issue in many applications. Besides locating equilibrium states, it is often also important to identify the transition states given by saddle points. In this talk, we will discuss numerical algorithms, in particular, the shrinking dimer dynamics, for the computation of such transition states and present some recently developed mathematical theory. We will consider a number of applications including the studies of interacting particles on unit spheres and critical nuclei morphology in solid state transformations.

## Modeling rare events

**Weinan E**

*Peking University and Princeton University*

Many important processes in nature are rare events. Familiar examples include conformational changes of bio-molecules, nucleation events and chemical reactions. Other examples may include the collapse of the stock market, and the spreading of infectious diseases. From an abstract viewpoint, this can be formulated as the problem of navigating a system over its energy landscape. For simple systems, the classical transition state theory in chemical physics and large deviation theory in mathematics provide an effective description for the transition pathways and rates of such rare events. For systems with complex energy landscapes, however, one needs a new language and new framework to describe such transition events. We will discuss a new theory, the transition path theory, that is more suited for describing transitions in complex systems. We will also discuss the string method for finding transition pathways and transition rates, that has proven to be quite effective for a wide variety of problems. We will also discuss applications to a variety of problems in science and engineering.

## Interface problems for two-fluid mixtures

**Aaron Fogelson**

*University of Utah*

I will discuss approaches to solving two kinds of interface problems for two-fluid (solvent-network) mixture models. In these models both fluids may be present at each point in space and there are separate velocity fields for the two fluids. One problem is concerned with the situation in which a region containing a mixture of the two fluids (a 'gel') is adjacent to a region with only solvent. The equations become degenerate in the pure-solvent region. We introduce an interface-capturing method based on regularizing the mixture equations so as to be able to solve them throughout the domain. The second problem involves an actual boundary which interacts with the two-fluid mixture. We consider a deformable elastic boundary and introduce a new Immersed Boundary method for solving this problem.

## **Phase field model of Thermo-Induced Marangoni effects and Marangoni-Rayleigh-Benard instability in a two-fluid mixture using a finite element method**

**Zhenlin Guo**

*University of Science and Technology Beijing*

We will derive a phase field model for a two-fluid mixture where the fluid density and surface tension depend on the temperature. The thermal induced Marangoni effects of these two Newtonian fluids, especially thermal induced instability at their interface will be considered. We shall adopt an appropriate variational form and a continuous finite element method which may maintain any possible underlying energy law to its greatest extent. Instability at a relatively simple interface will be studied theoretically and numerically.

## **A multigrid method for solving implicit-time immersed boundary equations**

**Robert Guy**

*University of California at Davis*

The immersed boundary method originally developed to solve the coupled equations of motion of a viscous, incompressible fluid and one or more massless, elastic surfaces or objects immersed in the fluid. The method involves two coordinate

systems and two discrete grids. The fluid variables are represented in Eulerian coordinates which are discretized by a fixed, Cartesian grid. The immersed structures are represented in moving Lagrangian coordinates. Typical implementations of the IB method use a fractional stepping approach to solve the coupled fluid and boundary equations. The fluid velocity and pressure are updated for fixed boundary position, and then the boundary position is updated from the new velocity. Because the fluid and boundary are updated separately, one can use standard methods for solving for the fluid motion. One reason for the popularity of the IB method is that many different applications can be simulated with minor changes to existing codes. However, in many applications the elastic time scales are well below the physical time scales of interest, which means that the IB equations are very numerically stiff.

Implicit time discretizations are needed to remove the stiffness, but at the cost of solving the equations on the Eulerian and Lagrangian grids simultaneously. This makes it difficult to use fast methods that exploit the regular structure of the Eulerian grid. In this talk a multigrid method for solving the linearized immersed boundary equations that arise in implicit time discretizations is presented. Numerical tests that compare the efficiency of the method with an explicit-time method are presented for a variety of test problems, including different geometries, variable numbers of Lagrangian points, and different constitutive laws. Analytical results are presented for simplified model problems to provide insight into the success and limitations of the method. Solving the implicit equations is only slightly more computationally expensive than solving the explicit equations, and time dependent simulations are up to 100 times more efficient. By using the multigrid method as a preconditioner for a Krylov method, the robustness of the method is extended and the efficiency of the computations is even greater for very stiff problems.

## **A family of models in X-ray dark-field tomography**

**Weimin Han**

*Department of Mathematics University of Iowa, Iowa City, IA 52242, USA*

X-ray mammography is currently the most prevalent imaging modality for screening and diagnosis of breast cancers. However, its success is limited by the poor contrast between healthy and diseased tissues in the mammogram. A potentially prominent imaging modality is based on the significant difference of x-ray scattering behaviors between tumor and normal tissues. Driven by major practical needs for better x-ray imaging, exploration into contrast mechanisms other than attenuation has been active for decades, e.g., in terms of scattering, which is also known as dark-field tomography. In this talk, a theoretical study is provided for the x-ray dark-field tomography (XDT) assuming the spectral x-ray detection technology.



The radiative transfer equation (RTE) is usually employed to describe the light propagation within biological medium. It is challenging to solve RTE numerically due to its integro-differential form and high dimension. For highly forward-peaked media, it is even more difficult to solve RTE since accurate numerical solutions require a high resolution of the direction variable, leading to prohibitively large amount of computations. For this reason, various approximations of RTE have been proposed in the literature. A family of differential approximations of the RTE is introduced and analyzed.

## **A hybrid simulation of large-eddy simulation and immersed boundary method for bio-locomotion at moderate Reynolds numbers**

**Guowei He**

*LNM, Institute of Mechanics, Chinese Academy of Science*

Numerical simulation of flapping wings at moderately high Reynolds numbers presents two challenges to computational fluid dynamics: turbulent flows and moving boundaries. On the one hand, direct numerical simulation of turbulent flows is not feasible in practice due to current capability of computer and the Reynolds averaging Navier-Stokes method is difficult in predicting the unsteady properties of turbulent flows. Instead, large-eddy simulation, which simulates the motions of large-scale eddies with modeling the effects of small-scale eddies on large-scale ones, becomes a powerful simulation technique for turbulent flows; On the other hand, moving boundaries usually need body-fitting grids and the grid generation is time-consuming. Immersed boundary method uses non-body-fitting grids which can treat the moving boundaries on Cartesian grids efficiently. Therefore, a combination of large-eddy simulation and immersed boundary method is promising that can efficiently simulate the unsteady aerodynamics of flapping wings. In this talk, I will introduce our recent work on how to combine large-eddy simulation and the immersed boundary method. The hybrid approach is used to simulate a SD7003 airfoil in plunge and/or pitch motions at  $Re=60,000$ . The results obtained are compared with the previous numerical simulations and experimental measurements. Finally, I will present our large-scale simulation for bat flight, using 100 million grids.

## **The minimal finite element method for the linear elasticity problem within the stress-displacement formulation**

**Jun Hu**

*School of Mathematical Sciences, Peking University*

Surprisingly a pair of the finite element spaces with the minimal degrees of freedom can form a stable and convergent discrete scheme for the linear elasticity problem within the stress-displacement formulation for any space dimension. There are the simplest elements and can not be improved. In addition, the superconvergence property is observed for this family of elements.

## **The compact discontinuous Galerkin method for nearly incompressible linear elasticity**

**Jianguo Huang<sup>a</sup>      Xuehai Huang<sup>b</sup>**

<sup>a</sup>*Department of Mathematics, Shanghai Jiao Tong University, Shanghai 200240, China  
Division of Computational Science, E-Institute of Shanghai Universities, Shanghai Normal University, China*

<sup>b</sup>*College of Mathematics and Information Science, Wenzhou University, Wenzhou 325035, China*

A compact discontinuous Galerkin method (CDG) is devised for nearly incompressible linear elasticity, through replacing the global lifting operator for determining the numerical trace of stress tensor in a local discontinuous Galerkin method (LDG) by the local lifting operator and removing some jumping terms. It possesses the compact stencil, that means the degrees of freedom in one element are only connected to those in the immediate neighboring elements. Robust optimal error estimates in broken energy norm,  $H^1$ -norm and  $L^2$ -norm are derived for the method, which are uniform with respect to the Lamé constant  $\lambda$ . Furthermore, we obtain a post-processed  $H(\text{div})$ -conforming displacement by projecting the displacement and corresponding trace of the CDG method into the Raviart-Thomas element space, and obtain robust optimal error estimates of this numerical solution in  $H(\text{div})$ -seminorm and  $L^2$ -norm. Some numerical results are provided to illustrate the numerical performance of our method.

## **Finite element analysis of electromagnetics in metamaterials**

**Yunqing Huang**

*Xiangtan University, China*

In this talk we will report some recent advances in finite element analysis of electromagnetics in metamaterials. The stability properties, optimal error estimates and superconvergence are considered for various fully discrete schemes. Numerical tests are presented both for theoretical justification and some typical phenomenon such as cloaking, backward wave propagation etc.

This talk is based on the joint work with Jichun LI and Wei YANG

## Nonsmooth Optimization, Theory and Applications

**Kazufumi Ito**

*North Carolina State University*

We develop a Lagrange multiplier theory for Nonsmooth optimization, including  $L^\infty$  and  $L^1$  optimizations,  $\ell^0$  (counting metric) and  $L^0$  (Ekeland metric), Binary and Mixed integer optimizations and Data mining. A multitude of important problems can be treated by our approach and numerical algorithms are developed based on the Lagrange multiplier theory. and Semi-smooth Newton method.

## Asymptotic-Preserving schemes for kinetic-fluid modeling of two phase flows

**Shi Jin**

*Shanghai Jiao Tong University, China and University of Wisconsin-Madison, USA*

We consider systems coupling the compressible or incompressible Navier-Stokes equations to the Vlasov-Fokker-Planck equation. Such a problem arises in the description of particulate flows. We design numerical schemes to simulate this system. These schemes are asymptotic-preserving, thus efficient in both the kinetic and hydrodynamic regimes. It has a numerical stability condition controlled by the non-stiff convection operator, with an implicit treatment of the stiff drag term and the Fokker-Planck operator. Yet, consistent to a standard asymptotic-preserving Fokker-Planck solver or an incompressible Navier-Stokes solver, only the conjugate-gradient method and fast Poisson and Helmholtz solvers are needed. Numerical experiments are presented to demonstrate the accuracy and asymptotic behavior of the schemes, with several interesting applications.

This is a joint work with T. Goudon, J.-G. Liu and Bokai Yan

# A sinc collocation method for Love's integral equation with a very small parameter

**Xiaoqing Jin**

*Department of Mathematics, University of Macau, Macao  
(joint with Fu-Rong Lin and Xin Lu)*

We introduce an efficient numerical method for solving Love's integral equation

$$f(y) + \frac{1}{\pi} \int_{-1}^1 \frac{c}{(x-y)^2 + c^2} f(x) dx = 1, \quad y \in [-1, 1],$$

where  $c > 0$  is a parameter, by using a sinc-collocation method based on the double exponential transformation. We propose our numerical method by using the property that the solution  $f(y)$  of Love's integral equation satisfies:  $f(y) \rightarrow 0.5$  for  $y \in (-1, 1)$ , when the corresponding parameter  $c \rightarrow 0$ . Numerical results show that the proposed method is very efficient.

# A posteriori error analysis of finite element methods for linear nonlocal diffusion and peridynamic models

**Lili Ju**

*University of South Carolina*

In this talk, we present some results on a posteriori error analysis of finite element methods for solving linear nonlocal diffusion and bond-based peridynamic models. In particular, we aim to propose a general abstract framework for a posteriori error analysis of the peridynamic problems. A posteriori error estimators are consequently prompted, the reliability and efficiency of the estimators are proved. Connections between the a posteriori error estimations of the nonlocal problems and that of the related classical partial differential equation based problems are studied within continuous finite element spaces. Some numerical experiments are also given to test the theoretical conclusions.

# The ghost solid method for the elastic solid-solid interface

Kaboudian A,      Khoo BC

*Department of Mechanical Engineering National University of Singapore Kent Ridge,  
Singapore 119260*

Dynamics of elastic solid-solid interactions and the study of wave propagation through elastic mediums can be found in many diverse applications including but not limited to geophysical dynamics, biomedical research, non-destructive testing and others. In this work, three variants of Ghost Solid Method are proposed for application to the boundary conditions at the solid-solid interface of isotropic, linearly elastic materials, in a Lagrangian framework. In the presence of wave propagation through the solid-solid mediums, a scheme for prediction of non-physical oscillations at the interface is also introduced. Of the three variants proposed, it is found that two variants of the Ghost Solid Method developed can successfully remove the non-physical oscillations that may rise at the interface. Although the development is carried out primarily under one-dimensional setting, the approach can be easily extended to multi-dimensional settings for slip and no-slip conditions at the interface. Numerous numerical examples in one and two dimensional settings are provided attesting to the viability and effectiveness of the Ghost Solid Method for treating wave propagation at the solid-solid interface.

## Dynamics of supercavitating flow: Use of a Homogenous flow method

Khoo B.C.<sup>ab</sup>      Zheng J.G.<sup>a</sup>

<sup>a</sup> *Department of Mechanical Engineering, National University of Singapore, Singapore  
119260*

<sup>b</sup> *Singapore-MIT Alliance, National University of Singapore, Singapore 117576*

A numerical study on the simulation of supercavitation is presented based on a compressible Navier-Stokes flow solver. The supercavitating flow is treated as the homogeneous mixture of pure liquid water and vapour, while the cavitation inception and evolution are modeled by isentropic cavitation model. The governing equations are numerically integrated on unstructured triangular mesh using cell-centered finite volume MUSCL scheme. The solver is validated against experimental data and proves to be accurate and robust. The simulation for case of supercavitating flow around cylinder with blunt head is carried out to investigate skin friction drag reduction with supercavity evolution over the object. The variations of the physical quantities like lift exerted on the body and of the main flow

features associated with different angles of attack are reported and discussed. In addition, small cavitator is introduced to the object in an attempt to control the supercavity shape and reduce form drag on the body, and the simulation results are presented.

## **Microbial-induced mineralization in biofilms**

**Isaac Klapper,      Tianyu Zhang**

*Department of Mathematical Sciences & Center for Biofilm Engineering, Montana State University*

The familiar view of microbes in their free (planktonic) state is not the norm; rather it is believed that much of the microbial biomass, perhaps 95-99%, is located in close-knit communities, designated biofilms and microbial mats, consisting of large numbers of organisms living within self-secreted matrices constructed of polymers and other molecules. (Microbes in collective behave very differently from their planktonic state; even genetic expression patterns change.) These matrices serve the purposes of anchoring and protecting their communities in favorable locations while providing a framework in which structured populations can differentiate and self-organize.

One of the salient features of biofilms is their spatial heterogeneity; they are not uniform, well-mixed systems like many laboratory microbial communities are. Because of spatial variation, advective and diffusive processes become influential. Further, when ionic quantities are important, these processes in turn can lead to electric field effects becoming significant. These issues are discussed in the context of a particular phenomenon, namely mineralization resulting from biological activity in biofilms.

## **New adaptive artificial viscosity method for hyperbolic systems of conservation laws**

**Alexander Kurganov**

*Tulane University*

We propose a new finite volume method for solving general multidimensional hyperbolic systems of conservation laws. Our method is based on an appropriate numerical flux and a high-order piecewise polynomial reconstruction. The latter is

utilized without any computationally expensive nonlinear limiters, which are typically needed to guarantee nonlinear stability of the scheme. Instead, we enforce stability of the proposed method by adding a new adaptive artificial viscosity, whose coefficients are proportional to the size of the weak local residual, which is sufficiently large at the shock regions, much smaller near the contact waves, and very small in the smooth parts of the computed solution.

We test the proposed scheme on a number of benchmarks for both scalar conservation laws, one- and two-dimensional Euler equations of gas dynamics and the Saint-Venant system of shallow water equations. The obtained numerical results clearly demonstrate the robustness and high accuracy of the new method.

This is a joint work with Yunlong Chen, Minlan Lei and Yu Liu.

## **A stochastic sparse grid collocation method for stochastic optimal control problems**

**Hyung-Chun Lee**

*Department of Mathematics, Ajou University, Suwon, Korea 443-749.*

In this talk, we propose and analyze a stochastic collocation method for solving optimal control problems for elliptic partial differential equations with random coefficients and forcing terms. These input data are assumed to depend on a finite number of random variables. We prove existence of optimal solution and derive an optimality system. In the method, we use a Galerkin approximation in space and a sparse grid collocation in the probability space. We provide error estimates for fully discrete solution using an appropriate norm and analyze the computational efficiency. Computational evidence complements the present theory and show the effectiveness of the sparse grid stochastic collocation method.

## **Hybrid well-balanced WENO schemes with different indicators for shallow water equations**

**Gang Li**

*School of Mathematical Science, Qingdao University, Qingdao, Shandong 266071, P.R. China.*

**( This is a joint work with Changna Lu and Prof. Jianxian Qiu. )**

In [*J. Comput. Phys.* 229 (2010) 8105-8129], Li and Qiu investigated the hybrid weighted essentially non-oscillatory (WENO) schemes with different indicators for Euler equations of gas dynamics. In this continuation paper, we extend the method to solve the one- and two-dimensional shallow water equations with source term due to the non-flat bottom topography, with a goal of obtaining the same advantages of the schemes for the Euler equations, such as the saving computational cost, essentially non-oscillatory property for general solution with discontinuities, and the sharp shock transition. Extensive simulations in one- and two-dimensions are provided to illustrate the behavior of this procedure.

## **On the stability of a class of Rosenbrock methods and applications in solving parabolic problems**

**Wenyuan Liao**

*Department of Mathematics and Statistics, University of Calgary, 2500 University Drive, NW, Calgary, AB, T2N 1N4, Canada.*

It is well-known that the semi-discrete ordinary differential equation(ODE) system resulting from spatial discretization of a parabolic partial differential equation, for instance, the heat equation, is highly stiff. Therefore numerical methods with stiff stability such as implicit Runge-Kutta method and Implicit multistep Method are preferred to solve the ODE system. Rosenbrock method, a special subclass of implicit Runge-Kutta method, is efficient since it is iteration-free for nonlinear problem, but suffers from the drawback of order reduction, when applied to nonlinear parabolic problems. In this paper we aim to construct a numerical algorithm for solving nonlinear parabolic partial differential equation in 1D supplemented with Dirichlet and Neumann boundary conditions by combining compact finite difference scheme in spatial discretization with high-order strongly A-stable Rosenbrock method in time. We carefully designed the Rosenbrock method so the drawback of order reduction was successfully avoided. It has been shown that the proposed numerical algorithm is strongly A-stable and fourth-order accurate in both spatial and temporal dimensions. Several numerical experiments have been conducted to demonstrate the efficiency and accuracy of the new algorithm.

## **Uncertainty quantification, modeling and simulation for physical and biological flow problems**



**Guang Lin**

*Pacific Northwest National Laboratory*

Abstract: Experience suggests that uncertainties often play an important role in quantifying the performance of complex systems. Uncertainty quantification, modeling and large-scale simulation are required in many applications. In many of these complex problems, the solution of the corresponding stochastic PDEs requires treating effectively problems in high dimensional spaces. In this talk, we will present effective new ways of dealing with this curse-of-dimensionality. The generalized Polynomial Chaos, adaptive ANOVA decomposition, and some stochastic sensitivity analysis techniques will be discussed in some detail. We present demonstrative examples from fluid/solid mechanics, climate, crystal growth, flow and transport in randomly heterogeneous porous media, and biology.

## **Phase field model of Thermo-Induced Marangoni effects and instability in a two-fluid mixture using a finite element method**

**Ping Lin**

*Division of Mathematics, University of Dundee*

We will derive a phase field model for a two-fluid mixture where the fluid density and surface tension depend on the temperature. We will study the thermal induced Marangoni effects of these two Newtonian fluids, especially thermal induced instability at their interface will be considered. We shall adopt an appropriate variational form and a continuous finite element method which may maintain any possible underlying energy law to its greatest extent. Instability at a relatively simple interface will be studied theoretically and numerically. The talk is based on a joint work with Zhenlin Guo.

## **A balanced finite element method for singularly perturbed reaction-diffusion problems**

**Runchang Lin<sup>a</sup>      Martin Stynes<sup>b</sup>**

<sup>a</sup>*Department of Mathematical and Physical Sciences, Texas A&M International University, Laredo, TX 78041-1900;*

<sup>b</sup>*Department of Mathematics, National University of Ireland, Cork, Ireland.*

Consider the singularly perturbed linear reaction-diffusion problem  $-\varepsilon^2 \Delta u + bu = f$  in  $\Omega \subset \mathbb{R}^d$ ,  $u = 0$  on  $\partial\Omega$ , where  $d \geq 1$ , the domain  $\Omega$  is bounded with (when  $d \geq 2$ ) Lipschitz-continuous boundary  $\partial\Omega$ , and the parameter  $\varepsilon$  satisfies  $0 < \varepsilon \ll 1$ . It is argued that for this type of problem, the standard energy norm  $v \mapsto [\varepsilon^2 |v|_1^2 + \|v\|_0^2]^{1/2}$  is too weak a norm to measure adequately the errors in solutions computed by finite element methods: the multiplier  $\varepsilon^2$  gives an unbalanced norm whose different components have different orders of magnitude. A balanced and stronger norm is introduced, then for  $d \geq 2$  a mixed finite element method is constructed whose solution is quasi-optimal in this new norm. By a duality argument it is shown that this solution attains a higher order of convergence in the  $L_2$  norm. Error bounds derived from these analyses are presented for the cases  $d = 2, 3$ . For a problem posed on the unit square in  $\mathbb{R}^2$ , an error bound that is uniform in  $\varepsilon$  is proved when the new method is implemented on a Shishkin mesh. Numerical results are presented to show the superiority of the new method over the standard mixed finite element method on the same mesh for this singularly perturbed problem.

## Immersed finite element methods for planar elasticity interface problems

Tao Lin

*Department of Mathematics, Virginia Tech, Blacksburg, VA 24061*

We consider planar elasticity interface problems whose solution domains are formed by multiple elasticity materials separated by well-defined interfaces. Numerous methods, in either finite difference and finite element formulations, have been developed for solving elasticity problems efficiently and accurately. However, conventional finite element methods have to use meshes constructed according to the materials interfaces. This talk presents immersed finite element (IFE) methods for solving planar elasticity interface problems with interface independent structured Cartesian meshes. Basic features of linear and bilinear IFE functions, including the unisolvent property, will be discussed. While both methods have comparable accuracy, the bilinear IFE method requires less time for assembling its algebraic system. Our analysis further indicates that the bilinear IFE functions are guaranteed to be applicable to a larger class of elasticity interface problems than linear IFE functions. Numerical examples are provided to demonstrate that both linear and bilinear IFE spaces have the optimal approximation capability, and that numerical solutions produced by a Galerkin method with these IFE functions for elasticity interface problems also converge optimally in both  $L^2$  and semi- $H^1$  norms.

# Entropy satisfying methods for kinetic Fokker-Planck equations

**Hailiang Liu**

*Iowa State University*

Kinetic Fokker-Planck equations have been widely used to study statistical effects of various complex dynamics. In such models, the entropy structure is the main mechanism for the underlying distribution to evolve into some stable equilibrium patterns. In this talk I'll recount progress toward understanding the role of the entropy in the design of numerical methods to capture the long-time behavior. The present schemes are shown to satisfy three important properties: i) positivity preserving; (ii) equilibrium preserving; and iii) entropy satisfying. These ensure that the schemes provide a satisfying long-time behavior, thus underline the efficiency to preserve the large-time asymptotic. Applications include the FENE dumbbell model in polymeric fluids, and biological dispersal in population dynamics.

# Hybrid RKDG and WENO methods for hyperbolic conservation laws

**Liu Tiegang      Cheng Jian**

*School of Mathematics and Systems Science, Beihang University*

In this talk, we introduce a multi-domain hybrid method by combining Runge-Kutta discontinuous Galerkin (RKDG) methods and weighted essentially non-oscillatory (WENO) schemes. The method is non-conservative based on a third order RKDG method and a fifth order finite difference WENO scheme (WENO-FD). At the artificial interface of matching RKDG and WENO, special treatments are used to tackle with discontinuities such as shock waves and preserve high order accuracy for smooth solution as well. Theoretical analysis shows the hybrid method has high order accuracy for smooth solution and numerical results also demonstrate it is robust for slow and strong shock simulations. With the employment of a RKDG method near geometry boundaries and a WENO-FD scheme for the inner flow field, the multi-domain hybrid RKDG and WENO-FD method becomes more flexibility in handling various boundary conditions than the classical WENO-FD schemes and can reduce computation cost and memory requirement greatly than the traditional RKDG methods.

# Central-upwind scheme for the shallow water system with thermodynamics

Alina Chertock<sup>a</sup>

Alexander Kurganov<sup>b</sup>

Yu Liu<sup>b</sup>

*a. Department of Mathematics, North Carolina State University State University, Raleigh, NC, 27695, USA*

*b. Mathematics Department, Tulane University, New Orleans, LA 70118, USA*

We introduce a central-upwind scheme for one- and two-dimensional systems of shallow-water equations with horizontal temperature gradients (the Ripa system). The scheme is well-balanced, positivity preserving and does not develop spurious pressure oscillations in the neighborhood of temperature jumps, that is, near the contact waves. Such oscillations would typically appear when a conventional Godunov-type finite volume method is applied to the Ripa system, and the nature of the oscillation is similar to the ones appearing at material interfaces in compressible multifluid computations. The idea behind the proposed approach is to utilize the interface tracking method, originally developed in [A. CHERTOCK, S. KARNI, A. KURGANOV, *M2AN Math. Model. Numer. Anal.*, 42(2008), PP. 991-1019] for compressible multifluids. The resulting scheme is highly accurate, preserves two types of “lake at rest” steady states, and is oscillation free across the temperature jumps, as it is illustrated in a number of numerical experiments.

## Numerical simulation of current driven sediment transport processes and resulting bedforms

Burkow Markus

*Institute for Numerical Simulation, University of Bonn Wegelerstrasse 6, 53115 Bonn, Germany*

Current driven sediment transport causes the evolution of bedforms like dunes, ripples or scour marks. These bedforms are formed by the interaction of entrainment and deposition of sediment particles. In this study we use a numerical simulation of the three dimensional fluid flow and the simultaneous transport to reproduce these sediment processes. To solve the incompressible Navier-Stokes equations we use NaSt3D as fluid solver for incompressible two-phase flow problems in three dimensions. Fifth order WENO schemes are applied for spatial discretization. For temporal discretization we apply Runge-Kutta schemes up to third order. The free surface and therefore the impact of surface tension between both fluid phases is tracked with a level set technique. The main agent of bedforms

in sediment transport is bed load transport. A common way to model bed load and induced bedforms is the Exner equation. Balancing incoming and outgoing sediment mass leads to a new sediment surface height. Single phase examples like dunes and ripples as well as two-phase phenomena like splash erosion illustrate the large variety of sediment forms, which can be reproduced by this model.

## **A direction splitting algorithm for flow problems in complex/moving geometries**

**P. Minev**

*Department of Mathematical and Statistical Sciences, University of Alberta, Edmonton, Alberta Canada T6G 2G1*

An extension of the direction splitting method for the incompressible Navier-Stokes equations proposed in [1], to flow problems in complex, possibly time dependent geometries will be presented. The idea stems from the idea of the fictitious domain/penalty methods for flows in complex geometry. In our case, the velocity boundary conditions on the domain boundary are approximated with a second-order of accuracy while the pressure subproblem is harmonically extended in a fictitious domain such that the overall domain of the problem is of a simple rectangular/parallelepiped shape. The new technique is still unconditionally stable for the Stokes problem and retains the same convergence rate in both, time and space, as the Crank-Nicolson scheme. A key advantage this approach is that the algorithm has a very impressive parallel performance since it requires the solution of one-dimensional problems only, which can be performed very efficiently in parallel by domain-decomposition Schur complement approach. Numerical results illustrating the convergence of the scheme in space and time will be presented. Finally, the implementation of the scheme for particulate flows will be discussed and some validation results for such flows will be presented.

## **Towards a bottom-up approach to blood flow**

**Chaouqi Misbah**

*Univ. Grenoble 1/CNRS, LIPhy (Laboratoire Interdisciplinaire de Physique) UMR5588, Grenoble, F-38041, France*

Blood flow is a long-standing problem of biomedical importance; in particular, the understanding of the physical mechanisms impacting cardiovascular diseases (such as atherosclerosis, the leading cause of death in Europe and USA) is still an open issue. Physical factors play a major role but even after nearly two centuries of research they remain poorly understood due to the lack of knowledge of the blood behavior at the cellular level (at the scale of red blood cells (RBCs), platelets, etc.). Blood is a dense suspension of micron-sized RBCs and this complex microstructure renders modeling of blood flow very challenging within the traditional framework of describing fluid flow, i.e., the Navier-Stokes equations. To date, blood flow has been described by means of phenomenological continuum models that rely on various assumptions, which are both difficult to justify and to validate. Our goal is to study blood flow by taking explicitly into account blood elements. The method is based either on the boundary integral formulation or on the level set method.

Dynamics of biomimetic (vesicles and capsules) and biological entities (RBCs) under various flows will be described in the Stokes regime (and some in the Navier-Stokes one), and the current state of the art reported both for vesicles, capsules and RBCs. We shall consider single entities and their collective behaviors. Comparison between theory and experiments will also be provided.

## **Stability and convergence analysis of the nonlinear epitaxial growth model without slope selection**

**Zhonghua Qiao**

*Department of Applied Mathematics, Hong Kong Polytechnic University*

This work considers the numerical solution of the nonlinear fourth order parabolic equations for the epitaxial film growth without slope selection. The numerical simulations of the epitaxial film growth require long time computations, and therefore large time-stepping methods become necessary. In this work, two energy stable schemes will be proposed for solving the nonlinear diffusion equation, which has the dissipative mechanism in the energy law. The related stability, solvability and convergence of these numerical schemes will be analyzed. Relatively large time steps can be used in the simulation for the proposed numerical schemes.

## **The immersed interface method for axis-symmetrical geometries**

**Juan Ruiz Álvarez** <sup>a</sup>      **Zhilin Li** <sup>b</sup>

<sup>a</sup> *Universidad de Alcalá de Henares*

<sup>b</sup> *North Carolina State University, USA*

We have developed and analyzed the Immersed Interface Method (IIM) for elliptic interface problems with axis-symmetric geometries. The coefficient of the partial differential equation (PDE) can have discontinuous coefficient across an arbitrary but axis-symmetric surface (interface). The solution and the flux can be discontinuous across the interface. The discretization is done using the cylindrical coordinates. Thus we can treat some three dimensional interface problems using a two dimensional formulation. Using a level set function we approximate the position of the interface that may or may not be aligned with the underlying grid. A weighted least squares method is used in order to approximate the interface quantities starting from the level set function. In order to test the performance and accuracy of the method, some experiments are presented.

## **Some recent advances on phase-field models for multiphase complex fluids**

**Jie Shen**

*Purdue University*

I shall present some recent work on phase-field model for multiphase complex fluids. Particular attention will be paid to develop models which are valid for problems with large density ratios and which obey an energy law.

I shall present efficient and accurate numerical schemes for solving the coupled nonlinear system for the multiphase complex fluid, in many case prove that they are energy stable, and show ample numerical results which not only demonstrate the effectiveness of the numerical schemes, but also validate the flexibility and robustness of the phase-field model.

## **Solving two-phase flows with interfaces and irregular domains by an IIM**

**Zhijun Tan**

*Department of Scientific Computing and Computer Science, Sun Yat-sen University,  
Guangzhou 510275, China*

In this talk, an immersed interface method (*IIM*) for solve incompressible two-phase flows involving interfaces on irregular domains is presented. Two sets of augmented variables are introduced to satisfy the boundary condition for the velocity and the continuity condition of the velocity across the interface. The augmented variables and/or the forces along the interface/boundary are related to the jumps in both pressure and velocity and the jumps in their derivatives across the interface/boundary and applied to the uid through jump conditions. The resulting augmented equation is a couple system of these two sets of augmented variables, and the direct application of the *GMRES* is impractical due to larger iterations. In this work, the decoupling of two sets of the augmented variables is proposed, and the decoupled augmented equation is then solved by the LU or the *GMRES* method. The discretized fluid equations incorporating the jump contributions on a staggered Cartesian grid are solved by the fast solver. The numerical results show that the overall scheme is second order accurate.

## **Spectral deferred correction methods on general quadrature nodes**

**Tao Tang**

*Hong Kong Baptist University*

In this talk, we will discuss Spectral Deferred Correction (SDC) methods on general quadrature nodes. It has been demonstrated that spectral deferred correction methods can achieve arbitrary high order accuracy and possess good stability properties. There have been some recent interests in using high order Runge-Kutta methods in the prediction and correction steps in the SDC methods, and higher order rate of convergence is obtained provided that the quadrature nodes are uniform. The assumption of the use of uniform mesh has a serious practical drawback as the well-known Runge phenomenon may prevent the use of reasonably large number of quadrature nodes. In this talk, we will provide a convergence analysis for the SDC methods based on the integral operators. Following the analysis, we are able to propose a modified SDC methods with high order integrators which can yield higher convergence rates on both uniform and non-uniform quadrature nodes.



# Introduction to decadal predictions by FGOALS-g2

Bin Wang<sup>a,b</sup>

Mimi Liu<sup>a</sup>

Yongqiang Yu<sup>a</sup>

Lijuan Li<sup>a</sup>

*a. State Key Laboratory of Numerical Modeling for Atmospheric Sciences and Geophysical Fluid Dynamics (LASG), Institute of Atmospheric Physics, Chinese Academy of Sciences*

*b. Ministry of Education Key Laboratory for Earth System Modeling, Center for Earth System Science, Tsinghua University*

This study designs the initialization and decadal prediction schemes of the Grid-point Version 2 of Flexible Global Ocean-Atmosphere-Land System Model (FGOALS-g2), a state-of-art coupled climate system model, and performs sets of ensemble experiments of 10-year and 30-year hindcast using the FGOALS-g2 to examine the predictability of internal variations on decadal timescales. Prior to the experiments, a nudging-based initialization (simply 'ASSIM', hereinafter) of the upper-ocean state using historical observations of sea temperature and salinity is implemented to reduce the negative impact of initial uncertainty on hindcasts. The correctness of ASSIM is evaluated by comparing its results with the observations and those from one of the historical runs for 20<sup>th</sup>-Century climate simulation. To alleviate the impact of model biases in sea temperature and salinity on the overall performance of the coupled model through nonlinear interactions among different processes and among different components, a dynamic bias correction scheme for decadal predictions is proposed, which preliminarily shows positive effects in presenting the climatology and mean annual cycle of SST. Preliminary evaluations on skill of FGOALS-g2 in decadal prediction show that with the ICs from the initialization and the newly proposed dynamic bias correction scheme, the model presents skillful hindcasts in variations of sea surface temperature (SST) and surface air temperature (SAT) on decadal timescale, particularly in the decadal variations of Niño 3.4 SST anomaly index and regional SAT anomaly in China. FGOALS-g2 well simulates/predicts the global warming and global ocean warming in both the historical run (referred to as 'CTRL') and HCST, but it overestimates these warming in CTRL. HCST reduces the above overestimations and presents the warming closer to the observed. A prediction for SAT of future 30 years is also analyzed in this study.

## Hybridizable discontinuous Galerkin method (HDG) for Stokes interface flow

Bo Wang<sup>a</sup>

Boo Cheong Khoo<sup>b</sup>

*a. Singapore-MIT Alliance, E4-01-04, 4 Engineering Drive 3, Singapore 117576*

*b. Department of Mechanical Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260. 1*

A hybridizable discontinuous Galerkin method for solving interfacial Stokes flow with variable surface tension is investigated. This method is based on a body fitted mesh such that the jump conditions across the interface can be naturally imposed. When the mesh is exactly fitted with the computational domain and interface, we proved that this method has optimal convergence rate for both velocity and pressure in  $L^2$ -norm. In the numerical experiments, body fitted meshes are produced by simply subdividing the interface elements along the interface in a uniform cartesian mesh, i.e. the interface are approximated by polygon with an error of  $O(h^2)$ . By this way, we simplify the mesh generation and numerical results show that second order convergence rate is obtained for both velocity and pressure

## **Global in time numerical stability for time dependent nonlinear PDEs**

**Cheng Wang**

*Mathematics Department; The University of Massachusetts; North Dartmouth, MA, USA*

Global in time stability for certain numerical schemes to time dependent nonlinear PDEs, such as incompressible fluid flow and bi-stable gradient system, are presented in this talk. For incompressible Euler and Navier-Stokes equation, a global bound in  $L^2$  norm for the numerical solution is obtained. For bi-stable gradient system, the convexity splitting nature of the numerical scheme assures its non-increasing energy, which in turn leads to a global bound in higher order Sobolev norms. Some long time numerical simulations will also be presented.

## **A fast finite difference method for three-dimensional space-fractional diffusion equations**

**Hong Wang**

*Department of Mathematics, University of South Carolina, Columbia, South Carolina 29208, USA*

Fractional diffusion equations model phenomena exhibiting anomalous diffusion that can not be modeled accurately by the second-order diffusion equations. Because of the nonlocal property of fractional differential operators, the numerical

methods for fractional diffusion equations often generate dense or even full coefficient matrices. Consequently, the numerical solution of these methods often require computational work of  $O(N^3)$  per time step and memory of  $O(N^2)$  for where  $N$  is the number of grid points.

In this talk we present a fast multistep finite difference method for space-fractional diffusion equations in three space dimensions. The method only requires computational work of  $O(N \log^2 N)$  per time step and memory of  $O(N)$ , while retaining the same accuracy and approximation property as the regular finite difference method with Gaussian elimination.

Our numerical example runs for three dimensional model problem of intermediate size show the following observations: To achieve the same accuracy, the new method has a significant reduction of the CPU time from more than 2 months consumed by a traditional finite difference method to 5.74 seconds for a problem with 36,000 grid points on a work station with 128GB memory. This demonstrates the utility of the method.

## Fast time-domain computation of wave scattering problems

**Li-Lian Wang**

*Division of Mathematical Sciences School of Physical and Mathematical Sciences Nanyang Technological University, 637371, Singapore*

Wave propagations in unbounded media arise from diverse applications. Intensive research has been devoted to frequency-domain simulation, e.g., for the time-harmonic Helmholtz problems and Maxwell's equations. Here, we are interested in time-domain computation, which is known to be more flexible in capturing wide-band signals and modeling more general material inhomogeneities and nonlinearities. In this talk, we shall show how to use tools in complex analysis to analytically evaluate circular and spherical non-reflecting boundary conditions (NRBCs), and how to efficiently deal with time-space globalness of such boundary conditions. Fast spectral-Galerkin solvers together with stable time integration and techniques for handling general irregular scatterers will be introduced for the simulation. We intend to demonstrate that the interplay between analytic tools, accurate numerical means and sometimes brute force hand calculations can lead to efficient methodologies for challenging simulations. This is a joint work with Bo Wang and Xiaodan Zhao.

## Active material systems and pattern formation

**Qi Wang**

*University of South Carolina and Nankai University*

Active material systems are abundant in nature and in novel functional materials. In this talk, I will discuss models for active materials and some barebone ingredients. I will then discuss the solution structure and dynamics of the system in a simple active liquid crystal model. Numerical simulation of the material system in 2- and 3-D settings will be presented with focus on pattern formation and temporal-spatial structure dynamics.

## **Fourier finite volume element method for solving global quasi-geostrophic equations**

**Quanxiang Wang<sup>a</sup>**

**Zhiyue Zhang<sup>a</sup>**

**Zhilin Li<sup>b</sup>**

<sup>a</sup>*Jiangsu Provincial Key Laboratory for Numerical Simulation of Large of Scale Complex Systems, School of Mathematical Sciences, Nanjing Normal University, Nanjing 210046, CHINA*

<sup>b</sup>*Center for Research in Scientific Computation & Department of Mathematics, North Carolina State University, Raleigh, NC 27695-8205, USA*

A new Fourier finite volume element method for solving quasi-geostrophic equations on a sphere has been developed in this paper. Using the spherical coordinates, a Fourier discretization is used in the longitudinal direction while a piecewise linear approximation is used in the latitudinal direction. In our proposed numerical method, the trial and test function spaces are carefully chosen. In addition, the pole singularity is eliminated successfully by changing the resolution near the pole. Some numerical experiments are presented to illustrate accuracy and efficiency of our method.

## **Analysis and simulation of wetting and contact angle hysteresis on rough surfaces**

**Xiao-Ping Wang**

*Department of Mathematics, Hong Kong University of Science and Technology*

We study the contact angle hysteresis on rough and chemically patterned surfaces from a phase-field model for immiscible two phase fluid. In the slow motion, the dynamic equations of the interface as well as the contact angle can be derived from the matched asymptotic expansions. The contact angle hysteresis can then be studied from these equations.

## **Weak Galerkin method for the Ciarlet-Raviart mixed formulation of biharmonic equations**

**Yanqiu Wang**

*Oklahoma State University*

This talk focuses on a weak Galerkin method for the Ciarlet-Raviart mixed formulation of biharmonic equation. It is a joint work with M. Lin, J. Wang, and X. Ye. The idea of the weak Galerkin method is to use a weakly defined gradient operator in the finite element discretization, which allows the use of totally discontinuous function spaces. Different from the discontinuous Galerkin method, the weak Galerkin method does not explicitly employ inter-element averages or jumps. The biharmonic problem is first rewritten into its Ciarlet-Raviart mixed formulation. Then, both the primal and the dual variables are discretized using equal-order weak Galerkin elements. Error estimations will be presented.

## **An unfitted HP-interface penalty finite element method for elliptic interface problems**

**Haijun Wu,      Yuanming Xiao**

*Nanjing University*

An  $hp$  version of interface penalty finite element method ( $hp$ -IPFEM) is proposed for elliptic interface problems in two and three dimensions on unfitted meshes. Error estimates in broken  $H^1$  norm, which are optimal with respect to  $h$  and sub-optimal with respect to  $p$  by half an order of  $p$ , are derived. Both symmetric and non-symmetric IPFEM are considered. Error estimates in  $L^2$  norm are proved by the duality argument. Numerical examples are provided to verify the theoretical findings.

# A coupled immersed interface and level set method for three-dimensional interfacial flows with insoluble surfactant

Jian-Jun Xu<sup>ad</sup>

Yunqing Huang<sup>ad</sup>

Ming-Chih Lai<sup>b</sup>

Zhilin Li<sup>c</sup>

<sup>a</sup> School of Mathematical and Computational Sciences, Xiangtan University, Xiangtan, Hunan, 411105, China

<sup>b</sup> Department of Applied Mathematics, National Chiao Tung University, Hsinchu 30050, Taiwan

<sup>c</sup> Department of Mathematics, North Carolina State University, Raleigh, NC 27695, USA & School of Mathematical Sciences, Nanjing Normal University, China

<sup>d</sup> Hunan Key Lab for Computation and Simulation in Science and Engineering, Xiangtan, Hunan, 411105, China

In this paper, the previous work on 2D interfacial flows with insoluble surfactant [J. Xu, Z. Li, J. Lowengrub, and H. Zhao, A level set method for solving interfacial flows with surfactant, *J. Comput. Phys.*, 212(2006)590 – 616.] is extended to the three-dimensional case. The numerical scheme consists of a 3D immersed interface method (*IIM*) for solving Stokes equations with jumps across the interface and a 3D level-set method for solving the surfactant convection-diffusion equation along an evolving deformable interface. A new modified *IIM* version for the 3D Poisson equation with jumps across the interface is developed which is similar in spirit to the one in literature but differs from the assumption that the jump conditions of the solution and the flux are implicitly captured by the data available in a neighborhood of the interface. Numerical example demonstrates that the *IIM* Stokes solver achieves second-order accuracy exactly. A 3D drop with insoluble surfactant under shear flow is investigated numerically by comparing the results with small deformation theory and studying the influences of different physical parameters.

## The immersed interface method for two-fluid flows

Sheng Xu

*Southern Methodist University*

I will present in this talk an implementation of the immersed interface method for simulating flows of two immiscible fluids with different density and viscosity. In the immersed interface method, a two-fluid flow is formulated as one set of governing equations subject to singular forces supported on the two-fluid interfaces; the flow is solved on a fixed Cartesian grid; and the jump conditions induced by the discontinuous fluid properties and the singular forces enter discretization schemes.

I will show the principal jump conditions, discuss the difficulties in implementing them, and provide a few options to overcome the difficulties. I will give numerical results and comparisons to demonstrate the accuracy, efficiency and stability of the method.

## **Weak Galerkin finite element methods for partial differential equations**

**Junping Wang,      Xiu Ye**

*Department of Mathematics and Statistics University of Arkansas at Little Rock Little Rock, AR 72204*

Newly developed weak Galerkin finite element methods will be introduced for solving partial differential equations. Finite element methods can be classified as two big groups: conforming and nonconforming methods, (or roughly speaking: continuous and discontinuous methods). Constructions of conforming elements are not trivial in many situations. An alternative approach is to use discontinuous functions to approximate the true solutions. However, for discontinuous functions, the strong derivatives are not well defined. The concept of weak Galerkin methods is to introduce well defined weak derivatives for discontinuous functions. As the results, the weak Galerkin finite element methods are simple, flexible and parameter independent. Allowing the use of discontinuous approximating functions on arbitrary shape of polyhedra makes the methods highly flexible in practical computation.

## **Modulus-based successive overrelaxation method for pricing American options**

**Junfeng Yin,      Ning Zheng**

*Dept. of Mathematics, Tongji University*

Since the Chicago Board Options Exchange started to operate in 1848, the trading of options has grown to tremendous scale and plays an important role in global economics. Various type of mathematical models for the prices of different kinds of options are proposed during the last decades, and the valuation of options has been topic of active research. Consider the Black-Scholes model for American option, a high order compact scheme with local mesh refinement is proposed and

analyzed. Then, Modulus-based successive overrelaxation method is taken for the solution of linear complementarity problems from discrete Black-Scholes American options model. The sufficient condition for the convergence of proposed methods is given. Numerical experiment further show that the high order compact scheme is efficient, and modulus-based successive overrelaxation method is superior to the classical projected successive overrelaxation method.

## **HLLC-based flow solver for the seven-equation compressible two-phase model and its implementation on GPU**

**Shan Liang,      Wei Liu,      Li Yuan**

*Institute of Computational Mathematics and Scientific/Engineering Computing, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China*

An improved numerical method which combines a HLLC-type approximate Riemann solver with the third-order TVD Runge-Kutta method is presented for numerical approximation of the two-pressure and two-velocity seven-equation model of compressible two-phase flow due to Saurel and Abgrall. Based on the idea proposed by Abgrall that “a multiphase flow, uniform in pressure and velocity at  $t=0$ , will remain uniform on the same variables during time evolution”, discretization schemes for the non-conservative terms and for the volume fraction evolution equation are derived in accordance with the adopted HLLC solver for the conservative terms. To attain high temporal accuracy, the third-order TVD Runge-Kutta method is revised in conjunction with the operator splitting technique in a robust way by virtue of reordering the sequence of operators. Numerical tests against several one- and two-dimensional compressible two-fluid flow problems with high density and high pressure ratios demonstrate that the proposed method is accurate and robust.

In order to solve large-scale problems efficiently, we implement this numerical method on graphics processing units (GPU) using CUDA. The parallel work is for each GPU thread to compute a mesh point, for which we use appropriate data structure to obtain high memory bandwidth, and utilize synchronization of various types to avoid excessive switches between GPU and CPU. We observe 31X speedup relative to a single-core CPU run. By using domain decomposition method, we run the code on 8 GPUs with MPI and Pthread. The parallel efficiency is quite encouraging.

## **Numerical methods of quasicrystals**



**Pingwen Zhang**

*School of Mathematical Sciences Peking University, Beijing, 100871*

Quasicrystals (QCs), with long-range order and non-crystallographic symmetry, is one kind of fascinatingly ordered structures between period structures (crystals) and disordered structures. The discovery of QCs changes the traditional concept of classifying structures into: crystals and non-crystals, and gives a strong impact on materials science, solid state chemistry, condensed matter physics and soft matter, both on basic experimental and theoretical tools. This concept of QCs also promotes the development of several branches of mathematics, such as number theory, geometry, group theory and applied mathematics.

The first observation of QCs was done in April 1982 by D. Shechtman. He observed that a rapid cooled Al-Mn alloy exhibit 5 fold non-crystallographic symmetry. Since the original discovery of D. Shechtman, hundreds of QCs have been reported and confirmed in metallic alloys with 5, 8, 10, 12 fold orientational symmetry. Shechtman also received the Nobel Prize in Chemistry in 2011 for the discovery of QCs. Two decades after the first discovery of QCs in metallic alloys, soft-matter QCs were found in nature. Since then a number of materials successively joined the family of soft QCs, including liquid crystals, polymers, nonoparticles, colloids and mesoporous silica.

There exist several difficult problems in the theoretical research on QCs: how to building mathematical models which make the QCs exist and thermodynamical stability; how to design general numerical methods to capture QCs; how to compare the theoretical results with physical experiments quantitatively. Here, we focus on the development of numerical methods. QCs are a kind of whole-spatial structures. Traditionally the same dimensional numerical methods need period structures to approximate QCs. However, these methods can just compute few kinds of QCs, such as 12-fold QCs, because of the limitation of Simultaneous Diophantine Approximation. We provide a systematic numerical method to calculate all QCs where QCs could be treated as projections of a higher-dimensional space. We also present how to compute the energy density exactly without boundary effect. Finally, we take Lifshitz-Petrich model as an example to demonstrate our methods and show some numerical results.

## **Resolving the boundary-layer on your laptop: A fourth-order projection method for incompressible flows with adaptive mesh refinement**

**Qinghai Zhang**

*University of California at Davis*

A projection method with fourth-order accuracy both in time and space will be presented for solving the incompressible Navier-Stokes equations on periodic and no-slip domains with adaptive mesh refinement and parallel computing. Spatial discretization employs classical finite volume stencils while temporal integration adopts a semi-implicit, L-stable additive Runge-Kutta (ARK) method which treats the non-stiff convection term explicitly and the stiff diffusion term implicitly. The resulting Poisson- and Helmholtz-type linear systems are solved with an efficient multigrid algorithm. The central difficulty on no-slip domains, i.e. the non-commutativity of the projection and Laplacian, is handled by solving for the Stokes pressure and add its gradient back to complete the evolution of the velocity. The well-posedness and stability of this approach is confirmed by a recent analysis on the bound of the Laplace-Leray commutator. Numerical results shows that for the accuracy to be  $10^{-8}$ , running on my laptop with the fourth-order method is faster than the fastest supercomputer in the world with a second-order method! This superior efficiency is further enhanced by adaptive mesh refinement for additional power of resolving fine structures. I will also discuss the generalization of this approach to complex multi-phase flows such as free-surface flows and fluid-structure interaction.

## **Superconvergence: Unclaimed Territories**

**Zhimin Zhang**

*Washington State University*

While the superconvergence of the Galerkin finite element method is well understood, we know very little, if any, about superconvergence for the finite volume method. In this talk, we will touch this "unclaimed" territory by discussing superconvergence phenomenon for a family of finite volume methods of any order.

## **Time domain interface methods for electromagnetic wave propagation in dispersive media**

**Shan Zhao**

*Department of Mathematics, University of Alabama, Tuscaloosa, AL, USA, 35487*

Dispersive media are ubiquitous in nature, such as in biological tissues, rocks, soils, ice, snow, and plasma. In such media, the dielectric permittivity is a function of

frequency so that a broadband electromagnetic wave will propagate and attenuate in a frequency dependent manner. The interaction of such a wave with a dispersive interface, which separates a dispersive medium and a non-dispersive medium, is a very complex electromagnetic process. In particular, the electromagnetic field discontinuity across the dispersive interface is known to be frequency-dependent or time-varying in time domain simulations. Based on the auxiliary differential equation approach, we will examine a dispersive interface problem with the Debye dispersion model. A novel mathematical formulation will be established to describe the regularity changes in electromagnetic fields at the dispersive interface. The resulting time-dependent jump conditions will then be numerically enforced via the matched interface and boundary (MIB) scheme. Some preliminary numerical results will be reported.

## A 3D immersed boundary method with application

**Jian Hao<sup>a,b</sup>**      **Luoding Zhu<sup>a</sup>**

<sup>a</sup>*Department of Mathematical Sciences and Center for Mathematical Biosciences Indiana University - Purdue University, Indianapolis, IN 46202, USA*

<sup>b</sup>*Department of Mathematics and Center for Research in Scientific Computation, North Carolina State University, Raleigh, NC 27695, USA*

Most algorithms of the immersed boundary method originated by Peskin are explicit when it comes to the computation of the elastic forces exerted by the immersed boundary to the fluid. A drawback of such an explicit approach is a severe restriction on the time step size for maintaining numerical stability. The authors have proposed an implicit immersed boundary method in two dimensions using the lattice Boltzmann approach. In this letter we report an extension of the method to three dimensions and its application to simulation of a massive flexible sheet interacting with an incompressible viscous flow.