TRANSONIC EVAPORATION WAVES IN A SPHERICALLY SYMMETRIC NOZZLE

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Abstract. We study the liquid to vapor phase transition in a cone shaped nozzle. Using the geometric method presented in [21, 22], we extend results on subsonic and supersonic evaporation waves in [10] to transonic waves. It is known that transonic waves do not exist if restricted solely to the slow system on the slow manifolds. Thus we consider the existence of transonic waves that include layer solutions of the fast system that cross or connect to the sonic surface. In particular, we are able to show the existence of evaporation waves that cross from supersonic to subsonic regions and evaporation waves that connect from the subsonic regions to the sonic surface and then continue onto the supersonic branch via the slow flow.

1. Introduction

In this paper we study the liquid to vapor phase transition in a radially symmetric cone shaped nozzle. The model considered in this paper was proposed by Fan [8, 9] under the assumption that the heat capacity of the fluid is high so the evaporation is mainly caused by the pressure change, not the temperature change. Fan's model consists of a viscose p–system that describes the motion of compressible liquid–vapor mixture and a reaction-diffusion equation that describes the liquid to vapor phase transition:

$$\begin{aligned}
&\rho_t + \nabla\cdot(\rho u) = 0,\\
&(\rho u)_t + \nabla\cdot(\rho(uu) + p(\lambda,\rho)I) = \eta_1\nabla\cdot(\nabla u + \nabla u^T) + \eta_2\nabla\cdot((\nabla\cdot u)I),\\
&(\lambda\rho)_t + \nabla\cdot(\lambda\rho u) = \frac{w(\lambda,\rho)}{\gamma} + \mu\nabla\cdot(\rho\nabla\lambda),
\end{aligned} \tag{1.1}$$

where $\rho > 0$ is the density of the fluid, $u \in \mathbb{R}^3$ the velocity vector of the fluid, $\lambda \in [0, 1]$ is the mass fraction of vapor. All the constants in (1.1) are small parameters. Among them η_1 is the shear viscosity of the fluid, η_2 is a linear combination of the shear and the volume viscosity coefficients that is related to dilation of the fluid, and μ is the diffusion coefficient. The pressure $p(\lambda, \rho)$ is a function of the density ρ and the mass fraction of vapor λ which satisfies the assumptions

$$p_\rho > 0\,,\quad p_{\rho\rho} > 0\,,\quad p_\lambda > 0\,,\quad p(\lambda, 0) = 0\,,\quad p(\lambda,\infty) = \infty. \tag{1.2}$$

A representative pressure function $p(\lambda, \rho)$ is given by

$$p(\lambda, \rho) = C(1 + \lambda)\rho^k,\ C > 0,\ k > 1, \tag{1.3}$$

which fulfills all the conditions in (1.2). The function w is defined as

$$w(\lambda, \rho) := (p(\lambda, \rho) - p_{eq})\lambda(\lambda - 1)\rho, \tag{1.4}$$