STANDING WAVES FOR PHASE TRANSITIONS IN A SPHERICALLY SYMMETRIC NOZZLE

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Abstract. We study the existence of standing waves for liquid/vapor phase transition in a spherically symmetric nozzle. The system is singularly perturbed and the solution consists of an internal layer where the liquid quickly becomes vapor. Using methods from dynamical systems theory, we prove the existence of the internal layer as a heteroclinic orbit connecting the liquid state to the vapor state. The heteroclinic orbit is reproduced numerically and is also shown numerically to be a transversal heteroclinic orbit. The proof of the existence of an exact standing wave solution near the singular limit is based on the geometric singular perturbation theory and is outlined in the paper.

1. INTRODUCTION

In this paper, we study the spherically symmetric flow of liquid fuel which mimics the fuel being injected into the cylinder of an internal combustion engine through a cone shaped nozzle. We investigate the evaporation of liquid fuel in the nozzle and show that under certain conditions a steady evaporation front can occur inside the nozzle so that the fuel emerging from the injection nozzle is in its vapor state. In this way, the fuel vapor can mix with the air more evenly to ensure a more complete combustion. A complete combustion will increase the fuel efficiency and reduce the emission of engine.

The governing equations for flows involving liquid/vapor phase transitions are:

$$\begin{aligned}
&\rho_t + \nabla\cdot(\rho\mathbf{u}) = 0,\\
&(\rho\mathbf{u})_t + \nabla\cdot(\rho(\mathbf{u}\mathbf{u}) + \mathbf{P}) = 0,\\
&(\lambda\rho)_t + \nabla\cdot(\lambda\rho\mathbf{u}) = \frac{w}{\gamma} + \nabla\cdot(\mu\rho\nabla\lambda),\\
&E_t + \nabla\cdot(\mathbf{u}E + \mathbf{u}\cdot\mathbf{P}) = \kappa\Delta T + L(T)\nabla\cdot(\mu\rho\nabla\lambda),\\
&\mathbf{P} = (p + (\frac{2}{3}\epsilon_1 - \epsilon_2)(\nabla\cdot\mathbf{u}))I - \epsilon_1(\nabla\mathbf{u} + \nabla\mathbf{u}^T),\\
&p_\rho > 0,\ p_{\rho\rho} > 0,\ p_\lambda > 0.
\end{aligned} \tag{1.1}$$

See [16]. The major symbols in the equations are as follows

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