

Correction on P.1 for backward forecast

We already have $(y_1, y_2, y_3, y_4, y_5, y_6) \stackrel{D}{=} (y_3, y_2, y_1, y_0, y_{-1}, y_{-2})$

Let $Z_1 = (y_1, y_2, y_3)^T$, $Z_2 = (y_3, y_2, y_1)^T$

& the best predictor for y_4 is

$$\hat{y}_4 = \mu_{y_4} + V_{y_4 Z_1} V_{Z_1 Z_1}^{-1} (Z_1 - \mu_{Z_1}) = 2 + 0.9 y_3$$

The best predictor for y_0 is

$$\hat{y}_0 = \mu_{y_0} + V_{y_0 Z_2} V_{Z_2 Z_2}^{-1} (Z_2 - \mu_{Z_2})$$

Since this is stationary process,

$$\mu_{y_4} = \mu_{y_0}, V_{y_4 Z_1} = V_{y_0 Z_2}, V_{Z_1 Z_1}^{-1} = V_{Z_2 Z_2}^{-1}, \mu_{Z_1} = \mu_{Z_2}$$

$$\text{So } \hat{y}_0 = 2 + 0.9 y_1 = 2 + 0.9 \times 4 = 5.6$$

$$\text{Similarly, } \hat{y}_{-1} = 2 + 0.9 \hat{y}_0 = 7.04$$

$$\hat{y}_{-2} = 2 + 0.9 \hat{y}_{-1} = 8.336$$