Lecture 4: Curvature Scale Space

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Curvature Scale Space

$$C(s) = (x(s), y(s)) \quad (1)$$

Curvature:

$$\kappa(s) = \frac{(\dot{x}(s)\ddot{y}(s) - \ddot{x}(s)\dot{y}(s))}{(\dot{x}(s)^2 + \dot{y}(s)^2)^{3/2}} \quad (2)$$

Successively blurred by convolving it with a 1-d Gaussian kernel of width σ, where the scale(σ) is increased at each level of blurring. Now: $C(s,\sigma) : \Re \times \Re \longrightarrow \Re \times \Re$

$$C(s,\sigma) = (x(s) * G(s,\sigma), y(s) * G(s,\sigma)) \quad (3)$$

Why Curvature?

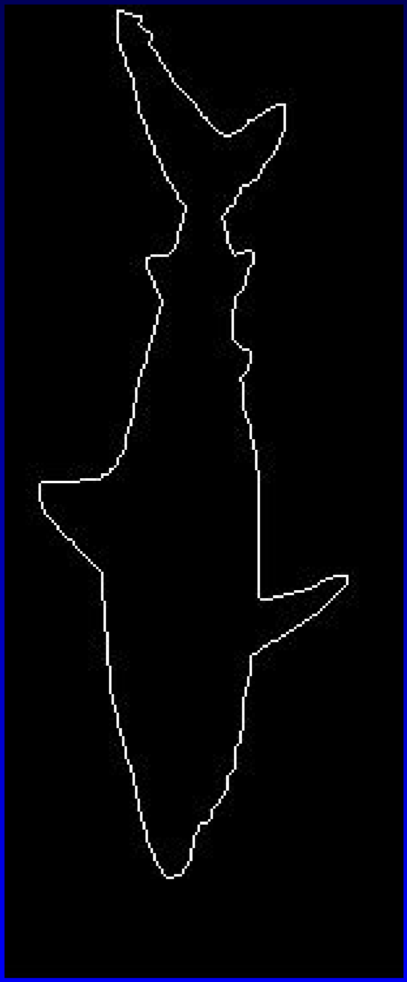
Invariant to viewpoint (rotation in the viewing plane, translation), but NOT zoom. Note:

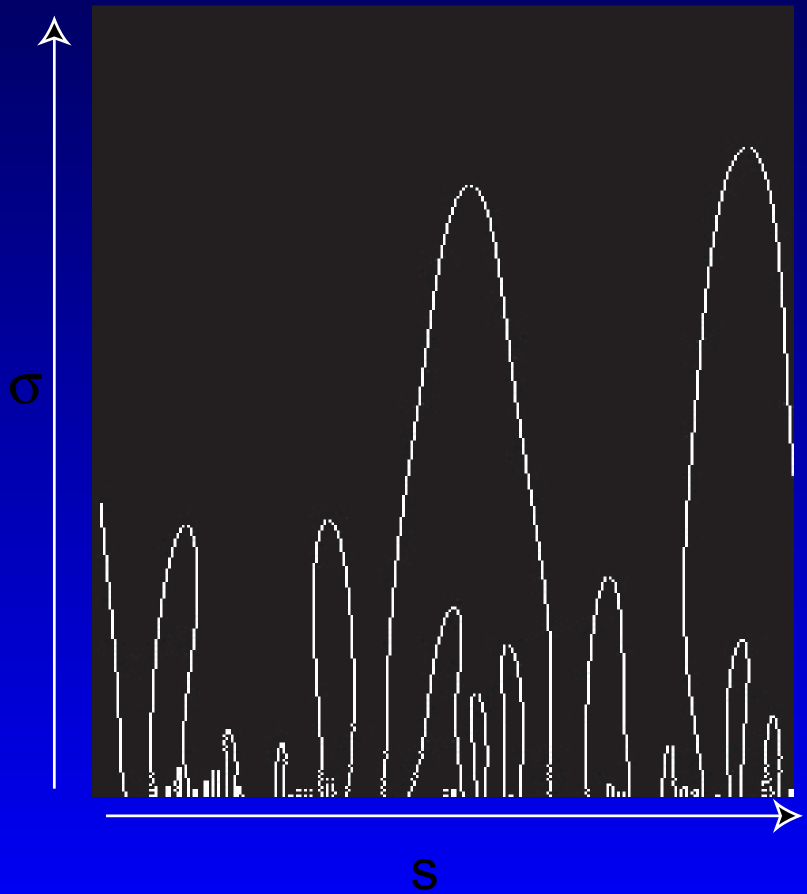
$$\kappa(s) = \frac{(\dot{x}(s)\ddot{y}(s) - \ddot{x}(s)\dot{y}(s))}{(\dot{x}(s)^2 + \dot{y}(s)^2)^{3/2}}$$

is invariant, but the Calculation/estimation of κ is NOT invariant, since it involved derivatives.

Curvature at each Scale

Curvature zero-crossings: points on the contour where the curvature changes sign.





Invariance

- translation: Curvature is a local property.
- rotation: Use of curvature in the viewing plane, translation)
- scale: (zoom) Accomplished by resampling the curve to a fixed perimeter

Note:

$$\kappa(s) = \frac{(\dot{x}(s)\ddot{y}(s) - \ddot{x}(s)\dot{y}(s))}{(\dot{x}(s)^2 + \dot{y}(s)^2)^{3/2}}$$

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Matching

Let $M_{target} = \{(t_1, \gamma_1), (t_2, \gamma_2), ..., (t_L, \gamma_L)\}$ be the maxima of the target, parametrized by arclength t and arranged in descending order of the scale γ. Let $M_{model} = \{(s_1, \sigma_1), (s_2, \sigma_2), ..., (s_N, \sigma_N)\}$ be the maxima of the model, parametrized by arclength s and arranged in descending order of scale σ. The matching algorithm is as follows:

Matching Algorithm

1. The highest maximum (with respect to the scale) in the target and the model CSS image are used to find the CSS shift parameter (correction for starting point reparameterization).

$$\alpha = s_1 - t_1 \tag{4}$$

   Two lists are initialized. One with the maximum pair (t_1, γ_1) from the target and another with the maximum pair (s_1, σ_1) from the model. The cost for the match is initialized as:

$$MC = |\sigma_1 - \gamma_1| \tag{5}$$

Matching Algorithm

2. For the next highest maximum (t_2, σ_2) in the target, apply the CSS shift parameter(α) calculated earlier. We find the closest maximum in the model to this shifted target maximum which is not present in the model list.

$$(s_i, \sigma_i) = \arg\min_{i=1}^{N} ||(s_i, \sigma_i) - (t_2 + \alpha, \gamma_2)||, (s_i, \sigma_i) \tag{6}$$

The two maxima are added to their respective lists.

Matching Algorithm

3. The cost of the match is updated as follows:

$$MC = \begin{cases} MC + ||(s_i, \sigma_i) - (t_2 + \alpha, \gamma_2)|| & \text{if } ||t_2 \\ MC + ||\gamma_2|| & \text{if } ||t_2 \\ MC + ||\gamma_2|| & \text{No m} \end{cases}$$

   where T is a user defined threshold.
4. Repeat 2 for all the elements in the target.

Matching Algorithm

5. Calculate the CSS shift parameter using the second highest maximum (t_2, γ_2) in the target and the model (s_2, σ_2) and also for maxima in the model which are close to the highest maximum of the target(within 80% of the maximum scale value). Repeat 1-3 for using these CSS shift parameters and calculate the match cost.
6. Repeat 1-4 by interchanging the place of the target and the model.
7. The lowest cost from all these matches is taken as the best match value.