An Introduction and Exploration of Metamaterials

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Abstract

Simply stated, metamaterials, or left-handed materials, have a negative index of refraction. Though counterintuitive, this can be achieved without breaking any laws of physics by building arrays of devices which respond to and act on electromagnetic waves. This paper will discuss the emerging state of technology of metamaterials. Where the technology currently is, where it is going, and what it will become will be addressed.

Introduction

In conventional materials, the index of refraction is ultimately controlled by the harmonics of the constituent atoms. These atoms may be modeled as a mass on a spring as follows.

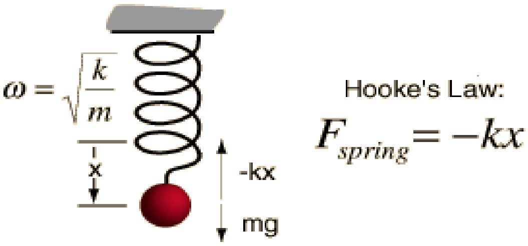

Figure 1. Mass on a spring resonance [Hyperphysics]

On a basic level, this is how indices of refraction are normally dependent on frequency. This same idea can be harnessed to engineer metamaterials. Homogeneous atomic materials are replaced by macroscopic inhomogeneous metals

The well understood Snell's law still applies to novel metamaterials, but as can be seen, when the index is changed to be negative, the bending of the light, as it exits a slab of material, will now be on the other side of the surface normal vector.

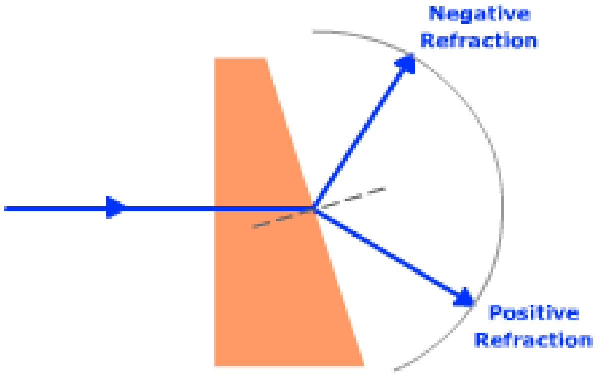

Figure 2 Comparison of Positive and Negative Index of Refraction Slabs [Smith]

$$n_1 \sin(q_1) = n_2 \sin(q_2)$$

This is now referred to as an inverted Snell's law. Now that there is an inverted Snell's law, there are unexpected characteristics in these left-handed metamaterials that can be demonstrated. Perhaps the most troubling of which is that apparently wave packets can now travel faster than light. The relation of the index of refraction to the speed of light can be seen by the following fundamental relations.

The impedance of a material is given by,

$$h = \sqrt{\frac{m_0 m_R}{e_0 e_R}}$$

and in free space such that

$$e_R = 1, m_R = 1, \; u = \frac{1}{\sqrt{m_0 e_0}} = c$$

the velocity of the wave is c. If the wave is not in free space, the permeability and permittivity of the material are also under the square root. With regards to the index of refraction,

$$n = \sqrt{\frac{me}{m_0 e_0}}.$$

At this point, it becomes apparent that if there is a negative index of refraction, there is a problem regarding u, the velocity of the wave through the material. A possible way around this fact is to say that it is only effectively true, but not necessarily physically true. The laws of physics are not broken by metamaterials. This paper will not attempt to explain this effect, and only strives to make the point apparent.

Another surprising characteristic of metamaterials is that the Doppler shift is reversed; a light source moving toward an observer appears to reduce its frequency. The phase and group velocities are in opposite directions of one another! This can also be shown using the above equations while examining β.

$$b = w\sqrt{me}$$

Cherenkov radiation results when a charged particle, most commonly an electron, exceeds the speed of light, represented by u in the above equation, in a dielectric. This radiation points the opposite direction in metamaterials from conventional materials. The Poynting vector is anti-parallel to phase velocity. This is resultant from the wave fronts moving in the opposite direction to the flow of energy. This does not mean that energy flows backwards to the source, but that the wave fronts go the “wrong” way.

Applications

An obvious military application of metamaterials is stealth technology and cloaking devices. It is potentially currently realizable to hide large items from some frequencies of radar. It would not be outlandish to assume that considerable research is taking place in this area.

Conventionally, optical systems are limited by the wavelength of a source light. The density of data that can be stored on a CD or Blu-Ray DVD is limited by the spot size of the wavelength of the laser used to read the data. The maximum resolvable features in photolithography steps of integrated circuit processing are also limited by the wavelength of the light source or laser. With metamaterials, it is proposed that image resolution can be arbitrarily high, which is an invaluable ability. Presumably, with a new sort of meta-microscopy, it is even possible to resolve atoms with light. It is debatable as to weather this could really be possible, but it gives rise to the question of what the limit of resolvability is when wavelength is no longer a limiting factor.

Construction of Metamaterials

It is interesting to note that opal (amorphous hydrated silicon dioxide gel with water content as high as 20%) is considered to be a natural metamaterial by some [Wikipedia, opal], while other sources note that no natural metamaterials exist. The metamaterial effects of opal is due to its macroscopic regular structure, and grain sizes, which are the proper order of size to interact with light. It is these macroscopic structures that give the material its special characteristics beyond the inherent atomic structure. This is precisely how artificial metamaterials are made.

When an understanding of the implications of a metamaterial has been gained, then structures can be engineered that create these effects. The crux of the topic is that a structure which is analogous to the unit constituents of a conventional refractive material must be created in such a way to respond to the desired frequency. It must also refract waves the desired amount. The majority of these devices rely on an application of the Biot-Savart law, or Ampere’s law of Maxwell’s equations. These two laws are closely related. The Biot-Savart law,

$$dH = \frac{Idl \times R}{4pR^3}$$

relates the magnetic field density, **B**, to the current flowing through some space. As a corollary, Ampere’s law,

$$\oint H \bullet dI = I_{enc}$$

relates the magnetic field though a closed loop of material, and in failure point form,

$$\nabla \times H = J \text{ .}$$

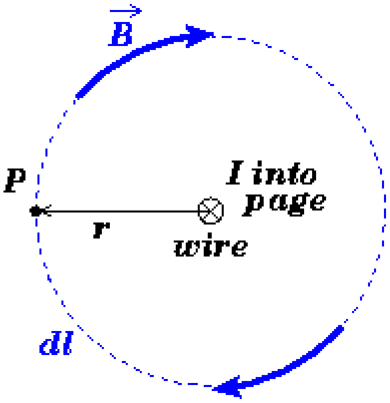

Figure 3. Ampere's Law [Gladney]

Specifically, when a magnetic field is applied to a conductor, a loop of wire, for example, a current will be inducted, as previously explained. This induced current flows in a path according to the right-hand rule generally in order to create its own opposing magnetic field to cancel out the applied magnetic field in the opposite direction. As it is, this is a good start to the problem introduced by metamaterials, to ensure that the resultant opposing magnetic field is much stronger than the applied field to not only cancel it out, but to reverse the vector entirely. To summarize, a magnetic anti-resonance is associated with an electric resonance.

The obvious approach to this was to create an array of unit cells that create magnetic moment in the other direction. These unit cells are rings of conducting metal arranged on a dielectric. The basic geometry of such a device is fabricated with microstrip technology in planar concentric rings. The split ring resonator (SRR) is an area of much focused research attention currently. By this method, Schurig et. al. have successfully hidden objects i.e. a copper cylinder within an invisible (to radar frequency) cloak. At even higher frequencies, up to 3THz, the SRR has been shown to be suitable.

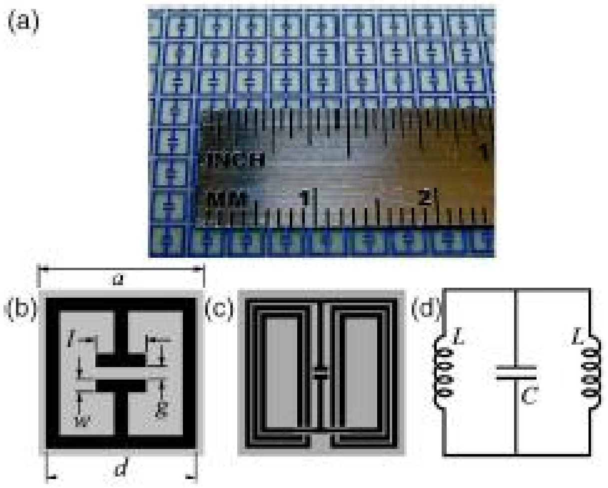

Figure 4. Split Ring Resonators [Schurig]

The size of these devices again is dependent on the wavelength it is designed to function on. For a unit cell of 30µm, a response of 1THz has been reached. To precisely select the spectrum the SRR is effective in, the rings may be tuned by slightly changing the geometry. The SRRs above designed by Schurig have inherent inductive-capacitive loads shown in 4.d, and are thus referred to as electric-LC resonators (ELC). These ELC resonators were able to attain a permittivity of -1+0.2*i*, as can be seen in the interesting data collected below.

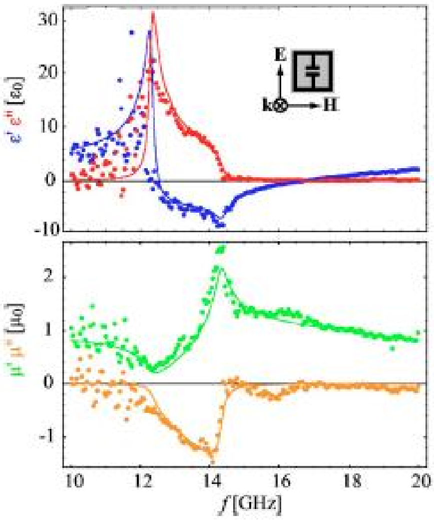

Figure 5. Frequency response of Permittivity and Permeability [Schurig, 9]

Another geometry useful to metamaterial fabrication, is the descriptively-named Swiss roll, as is illustrated below. Essentially it is a spiral electrolytic capacitor.

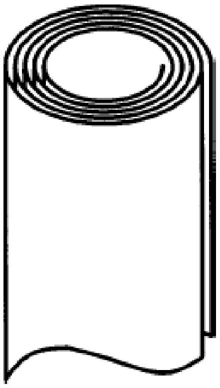

Figure 6. Swiss Roll Array Element [Wiltshire]

The Swiss roll elements in the system operate by the concept of self-inductance and thus posses a tunable resonance that can yield a high effective permeability (if designed properly). Again, this is all analogous to the harmonic oscillator model of atoms in a conventional material. Wiltshire et. al. showed that by choosing this particular geometry of conductor and dielectric, and building a repeating array, that the dielectric and magnetic properties can be tuned to create a metamaterial. The scale of this particular successful array was 271 units. The interesting property of Swiss rolls over SRR's is that they have an ability to induce a magnetic field in all of the other Swiss rolls in the array. This is due to the three-dimensional effects of the rolled metal as opposed to the two-dimensional SRR's.

Wilshire measured the current resulting from coupling to be managed by the equation,

$$\frac{I_0}{I_1} = 2\cosh((a + jk)D)$$

With the I's being the ratio of currents measured in the originating roll to the next, α a phase parameter, and D a coupling parameter. The measured response frequency was measured to be 21.9MHz.

Surface plasmons

Another approach to confine light beyond its wavelength limitation is reachable through surface plasmons. It is worth noting that the terms plasmoid and plasmon, are sometimes used interchangeably to describe what should properly be called a plasmon. A plasmoid is a coherent structure of plasma and magnetic fields [Wikipedia], which has an abrupt boundary. Surface plasmons are arguably a manifestation of the skin effect of conductors. This vibration of an electrical gas at optical frequencies is the vibration of the conduction band of metals. It is interesting to note that copper's color is due to the response frequency of the surface plasmons, while for most shiny silvery metals, the response is in the UV range. Stained glass and iridescence of butterfly wings is due to nanoparticles which created the desired surface plasmon frequency response. [Wikipedia] The below diagram illustrates this concept.

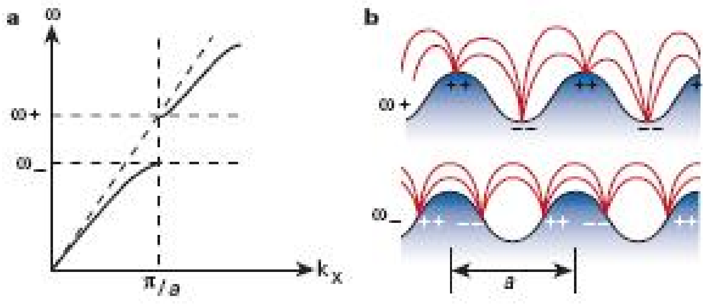

Figure 7.a and b Surface Plasmon bandgaps [Barnes]

Figure 7.a displays the relation of dispersion resultant from the texturization of surface plasmons as shown in figure 7.b.

These structures might also be described as nanowaveguides. An industrial interest in these effects is that these concepts can be applied to microwave panels with metal unit cells designed to operate at microwave frequencies. These can operate as filters or lenses. Due to Maxwell's equations, e. g. assuming a perfect conductor with σ = ∞, electric fields must be closed, and thus impinging electric field will be evanescent. That said, electric fields should not enter a metal, but they can interact on the surface. The effects of technology involving surface plasmons is similar to metamaterials in that light may be confined tighter than what would be otherwise possible, with exciting applications to microscopy and lithography. A technology known as a plasmonic cover has been developed by Engheta [2] which has the intent to create an invisibility shield.

Future Ambitions

One of the major hurdles of engineering useful metamaterials is calculating the proper layout of the unit cells depending on the topology of the item to be hidden (or effect of the lens). This is a mathematical problem involving a transformation of coordinates [Schurig]. Through mathematical transformations, ray tracing becomes possible, as is illustrated for the following spherical cloak.

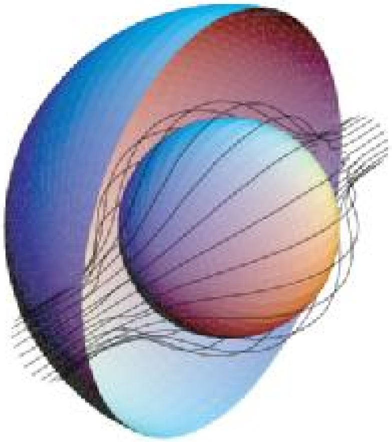

Figure 8. Spherical Cloak with Ray Tracing [Schurig]

Another inherent limitation is the frequency response of the device, which is generally very slim. As mentioned in the discussion on split ring resonators, the frequency response can be tuned by minor changes to the geometry of the unit cells. If a way to control the geometry of the unit cells could be easily physically changeable, a dynamic system could be built that can tune itself over a range of frequencies. As an extension to this idea, a system could be devised that fluctuates the tuning parameter faster than an observer would perceive. The result of this is a large effective bandwidth that a metamaterial could be responsive to. In the same way that a television screen scans faster than the human eye can perceive, so too could a cloaking system become invisible across a large portion of the visual spectrum. This is easier said than done, and is still very much science fiction.

The author proposes candidates as a means to this end could be transducers. The transducers could physically deform, unfold, or expand unit cells on a flexible material. Schurig has suggested that transistors be used to change the capacitances contained by the splits in the rings. Transistors could be used with a circuit to select to use rings of different sizes or geometry which are superimposed in the same areas by layers of laminate. There may be some way to dynamically change the micro-roughness of the conductors, having an effect on induction. A system using a combination of these tactics could benefit from the synergetic addition of all.

Conclusion

The theory of negative index of refraction metamaterials was explained in concept, starting with the basic concepts of Maxwell’s governing equations. Various devices built on this theory were described, including resonant split rings, and Swiss roll resonators. Surface plasmons were introduced because of the relevance of their similarity in function. The applications of metamaterials were also discussed, and by this, it has been shown that there is great value in further developing this technology. The applications of metamaterials can allow technology to perform a multitude of tasks that are not remotely possible today. The present short comings of metamaterials were discussed along with original ideas for diminishing them.

Admittedly, metamaterials is a subject that receives a monumental about of media hype, which can be deleterious to the interest level of physicists and engineers. However, to conclude that a subject is science fiction simply because it is cutting edge is an errant state of mind. It is a fact that these structures have been built and do perform with some degree of functionality. This technology is still in its early stages, but through more experimentation and diligent research, the payoff will be great.

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