



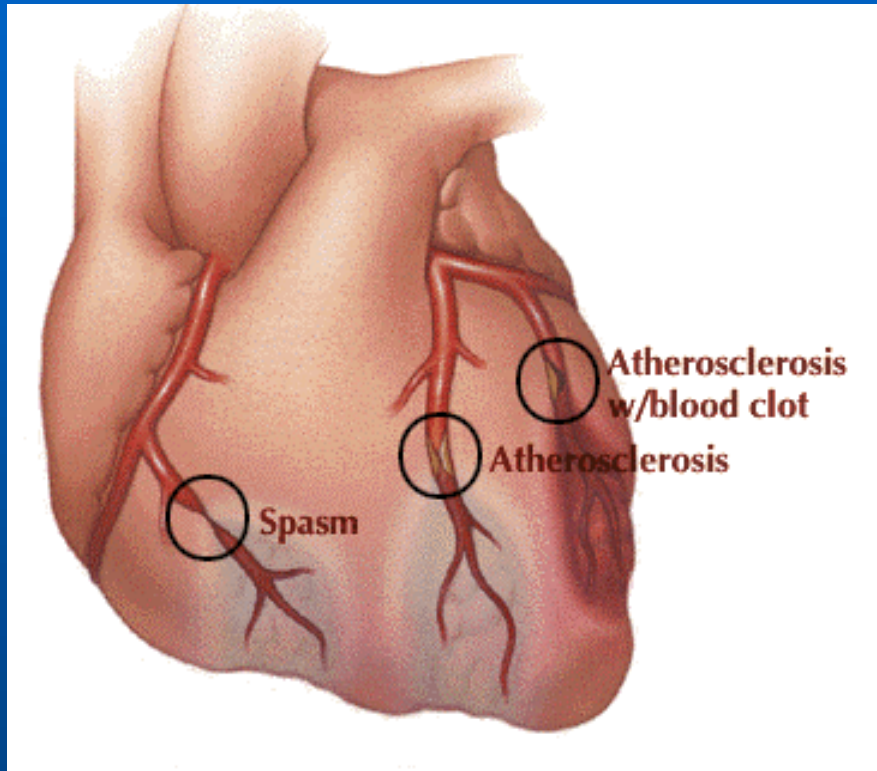
Wave Propagation



Experimental Set-Up



Atherosclerosis



- Build-up of plaque (fat and calcium deposits) on the interior walls of the arterial lumen
- The most common disease of arteries

Turbulence Induced Acoustic Waves

- **Significant deposits will disrupt the flow**

- Produce distinctive turbulence acoustic signatures



- **Large arteries (carotid arteries)**

- Detected by a stethoscope

- **Small arteries** deep inside the body, it is very difficult to detect

- Attenuation of acoustic energy through the intervening tissues
- Other complex sounds with the body can overwhelm the acoustic detection systems

Opportunity for CAD Detection

■ Unnecessary deaths

- 500,000 deaths each year (55% w/o warning)

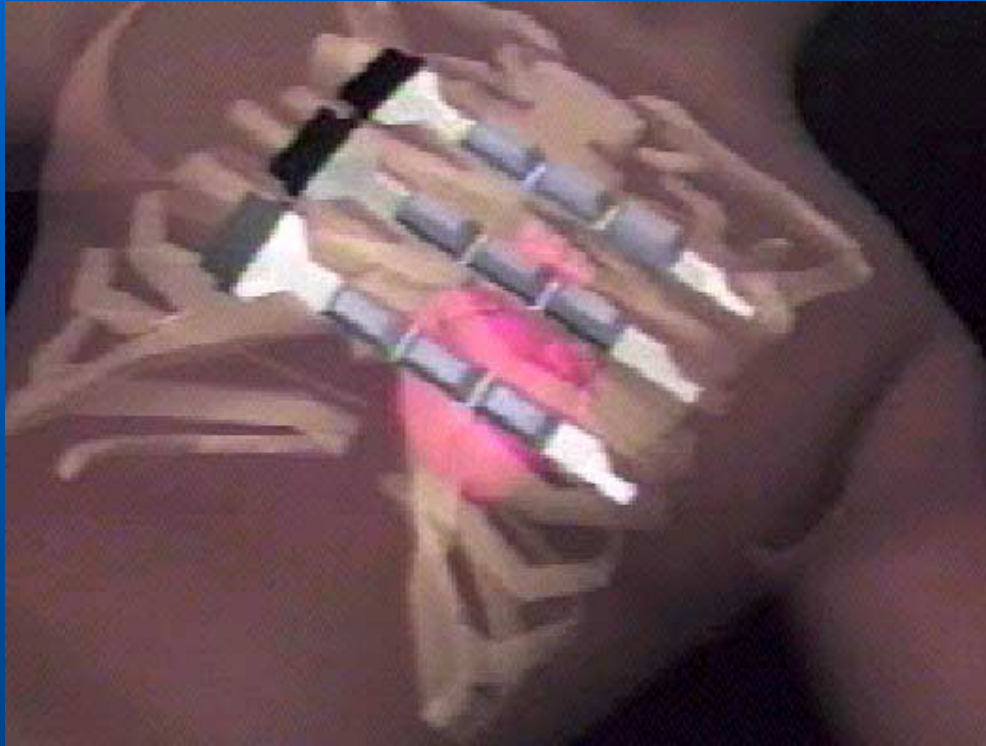
■ Unnecessary diagnostic tests

- 20-30% of angiogram patients are normal
- Costing \$2 billion each year

■ Unnecessary hospital admissions

- 600,000 chest pain patients/year admitted=normal
- Costing \$2 billion at \$4,000 avg. cost per day

MedAcoustics: Acoustic Sensors



- **Accurate**
- **Inexpensive**
- **Non-invasive**
- **Presents no risk to patient**
- **Easy to administer**

Wave Equation

- 1-D wave equation for the velocity potential:

$$\phi_{tt} = c^2 \phi_{xx}, \quad 0 < x < l$$

- Boundary conditions?

- Damped harmonic oscillator

$$m\delta_{tt} + d\delta_t + k\delta = -\rho\phi_t(t, 0)$$

- Boundary surface is not penetrable by the fluid

$$\delta_t(t) = \phi_x(t, 0)$$

Solution

$$\phi(t, x) = F(t - x/c) + G(t + x/c)$$

$$\delta_t(t) = \phi_x(t, 0) \quad \rightarrow \quad \delta(t) = -\frac{1}{c}(F(t) - G(t))$$

$$mG'' + (d + \rho c)G' + kG = mF'' + (d - \rho c)F' + kF$$

Assume incident wave is a simple harmonic

$$F(t - x/c) = e^{i\omega(t - x/c)}$$

$$G(t + x/c) = R(\omega)e^{i\omega(t + x/c)}$$

$$R(\omega) = \frac{m\omega^2 - i(d - \rho c)\omega - k}{m\omega^2 - i(d + \rho c)\omega - k}$$

An Inverse Problem

■ Problem:

Given $R(\omega_i)$ find the unknown parameters (m, d, k, ρ) in the damped harmonic oscillator boundary model

■ Determine $R(\omega)$?

- The acoustic pressure for a planar wave propagation is given by

$$p(t, x) = A(\omega)e^{i\omega(t-x/c)} + A(\omega)R(\omega)e^{i\omega(t+x/c)}$$

- By knowing $p(t, x_j)$ at a number of location x_j an inverse problem can be set-up to determine the complex functions, $R(\omega)$ and $A(\omega)$