

MA 293B
An Introduction to Applied Mathematics

PROJECT: ACOUSTIC WAVES IN A PVC PIPE

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The objective of this project is to study the effect of the boundary condition for acoustic waves propagation in a PVC pipe. Experiments with two different boundary conditions (hardwall and foam) will be carried out in the laboratory using a harmonic oscillator at the other boundary condition. These data are then used to obtain reflection coefficients over a wide range of frequencies. The reflection coefficients, in turn, are used to estimate unknown parameters in the model for the boundary condition.

The acoustic wave motion in a fluid is described by either the acoustic pressure, p , or the velocity potential, ϕ . These two quantities are related by $p(t, x) = \rho\phi_t$ and satisfy the wave equation

$$\phi_{tt} = c^2\phi_{xx}, \quad 0 < x < l \quad (1)$$

where c is the speed of sound, l is the length of the pipe, and the motion is assumed to be one dimensional. The following type of boundary condition will be considered:

Oscillating boundaries. The interaction of the boundary at $x=l$ and the interior pressure is modeled by a damped harmonic oscillator and is described by

$$m\delta_{tt} + d\delta_t + k\delta = -\rho\phi_t(t, l). \quad (2)$$

Here δ is the displacement (normal) of the boundary in the direction interior to the fluid. It is also assumed that the boundary surface is not penetrable by the fluid, that is,

$$\delta_t(t) = \phi_x(t, l). \quad (3)$$

We note that the solution to the wave equation (1) has the form

$$\phi(t, x) = F(t - x/c) + G(t + x/c) \quad (4)$$

where the first term on the right hand side of (4) describes a wave propagates to the right and the second term corresponds to a left propagating wave. Taking derivative of Eq. (4) with respect to x , substituting into Eq. (3), and integrating Eq. (3) with respect to t , we obtain

$$\delta(t) = -\frac{1}{c}(\tilde{F}(t) - \tilde{G}(t)) \quad (5)$$

where, without loss of generality, the constant of integration is set to zero and $\tilde{F}(t) = F(t - l/c)$, $\tilde{G}(t) = G(t + l/c)$. Substituting (5) into (2) yields

$$m\tilde{G}'' + (d + \rho c)\tilde{G}' + k\tilde{G} = m\tilde{F}'' + (d - \rho c)\tilde{F}' + k\tilde{F} \quad (6)$$

Now, assume that the incident wave \tilde{F} to the boundary at $x = l$ (which is generated by a harmonic input at $x = 0$), is a simple harmonic of frequency $\omega/2\pi$. That is,

$$\tilde{F}(t) = A_0 e^{i\omega t}, \quad (7)$$

so that the right hand side of (6) is a harmonic forcing function. It follows that the *steady state* solution of (6) is also harmonic with the same frequency

$$\tilde{G}(t) = R(\omega)A_0 e^{i\omega t}, \quad (8)$$

where the complex coefficient, $R(\omega)$, is called the *reflection coefficient*. Substituting equations (7) and (8) into (6) we have a relation for the reflection coefficient for the oscillating boundary condition model given by

$$R(\omega) = \frac{m\omega^2 - i(d - \rho c)\omega - k}{m\omega^2 - i(d + \rho c)\omega - k}. \quad (9)$$

This project involves the following steps:

1. The acoustic pressure anywhere in the pipe for planar wave propagation is given by the following equation:

$$p(t, x) = A(\omega)e^{i\omega(t-x/c)} + A(\omega)R(\omega)e^{i\omega(t+x/c)}$$

By measuring the pressure, $p(t, x_j)$, at a number of axial locations, x_j , and for a specific angular frequency $\tilde{\omega}$, an inverse least squares problem can be formulated to estimate both complex coefficients, $A(\tilde{\omega})$ and $R(\tilde{\omega})$. Considering both physical hardwall and foam type of boundary conditions at $x = l$, we will be collecting two corresponding sets of experimental data that one can use to estimate $R(\tilde{\omega})$. This data will be denoted by $R^d(\tilde{\omega})$, over the range of frequencies from 50 Hz to 500Hz. Note that $R^d(\tilde{\omega})$ will be given to you for both the physical hardwall and foam type of boundary conditions.

2. In this project, we will evaluate how well the oscillating boundary model described by formulas (9) fit the experimental data $R^d(\tilde{\omega}_j)$ (corresponding to a hard wall and soft wall

(that is, a foam)). That is, we will determine the set of parameters, (m, d, k, ρ) so that the functional

$$\sum_{i=1}^N |R^d(\omega_i) - R(\omega_i)|^2$$

is minimized. Here, N is the number of measurements R^d at frequencies $f_i = \omega_i / 2\pi$. In your report, discuss how good is the model (9) in describing the hardwall and the foam type of boundary condition.

3. You are also to turn in a one or two page summary of the module given by Prof. Tran.