

- 1.) Consider the following mathematical model for a spring-mass-dashpot system (using linear spring assumption, Hooke's law, and viscous air damping):

$$m \frac{d^2 y(t)}{dt^2} + c \frac{dy(t)}{dt} + ky(t) = 0$$

with initial conditions

$$y(0) = 1, \quad \frac{dy(0)}{dt} = 0,$$

and where m is the mass, c is the damping coefficient, k is the spring constant, and $y(t)$ is the vertical displacement of the mass about the equilibrium position.

The solution to the above second order differential equation can be computed using Matlab routine `ode45`. To use the routine `ode45` you need to rewrite the above equation as a system of first order differential equation. That is, let $x_1 = y$ and $x_2 = \frac{dy}{dt}$, we obtain

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= -\frac{k}{m}x_1 - \frac{c}{m}x_2 \end{aligned}$$

Let's assume that $m = 2$, $c = 2$, $k = 3$, and for $t \in [0, 5]$ compute the numerical solutions $x_1(t)$ and $x_2(t)$ and plot them on a graph.

- 2.) In general, the coefficients m , c , and k are unknown parameters. These parameters can be estimated as a nonlinear least squares estimation problem. That is, we seek $\vec{q} = [m, c, k]$ to minimize the cost function

$$J(\vec{q}) = \sum_{i=1}^{11} |x_2^m(t_i; \vec{q}) - x_2^d(t_i)|^2$$

where $x_2^m(t_i; \vec{q})$ is the solution to the spring mass dashpot model at time t_i for $i = 1, 2, \dots, 11$ given the parameter set \vec{q} and $x_2^d(t_i)$ is the data (speed) collected also at time t_i . In this exercise, we will create the "simulated" data to be used for estimating the unknown parameters $\vec{q} = [m, c, k]$. For this, we assume that displacement is sampled at equally spaced time intervals. We will subdivide the time interval $[0, 5]$ into 10 equal subintervals. That is, $t_1 = 0, t_2 = 0.5, t_3 = 1, \dots, t_{11} = 5$. Let $x_2^d(t_i)$ denote the displacement sampled at time t_i , $i = 1, \dots, 11$. This is the solution to the spring mass dashpot system corresponding to $m = 2$, $c = 2$, and $k = 3$. The idea is using these data, we will do an inverse problem of finding the parameters m , c , and k using the least squares criteria above. The solution to the above minimization problem can be solved using Matlab routine `fminsearch`. To use this subroutine you need to give a guess on the parameters (try $\vec{q}_g = [m, c, k] = [3, 1, 6]$ and $\vec{q}_g = [m, c, k] = [1, 1, 1]$). Create a table showing the guess values of the parameters \vec{q}_g , its cost function value $J(\vec{q}_g)$ and the optimal values of the parameters \vec{q}_o and its cost function value $J(\vec{q}_o)$. Also, plot the computed solution versus the data.