Sample Formula Sheet

- $$E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx,$$

  where $f_X(x)$ is the pdf of r.v. X, and $g(X)$ is a function of X. If X is a discrete r.v. taking values $x_j, j = 0, 1, 2, ...$, then

  $$E[g(X)] = \sum_{j=0}^{\infty} g(x_j) Pr(X = x_j).$$

- $$Var(X) = E[X^2] - (E[X])^2.$$

- $$S_X(x) = e^{-\int_0^x h_X(t)dt}.$$

- The pgf of X is $P_X(z) = E[z^X]$. It has the following properties:

  $$E[X] = P'(1); \quad E[X^2] = P'(1) + P''(1); \quad Pr(X = k) = \frac{P^{(k)}(0)}{k!},\ k = 0, 1, 2, ...$$

- mgf and pgf:

  $$M_X(t) = E[e^{tX}], \quad P_X(z) = M_X(\ln z), \quad M_X(t) = P_X(e^t).$$
  $$E[X^k] = M_X^{(k)}(t)|_{t=0}.$$

- $X_1, X_2, X_3, \cdots$ are independent, identically distributed (i.i.d.) r.v.'s and their pgf is given by $P_X(z)$. N is a r.v. which is independent of $X_1, X_2, \cdots$. Define a random variable S as

  $$S = X_1 + X_2 + \cdots + X_N.$$

  Then we have

  $$P_S(z) = P_N(P_X(z)).$$

- Unconditional pdf of X with a random parameter Θ:

  $$f_X(x) = \int_{-\infty}^{\infty} f_{X|\Theta}(x|\theta) f_\Theta(\theta) d\theta.$$

- $$E[X] = E[E[X|\Theta]], \quad Var(X) = E[Var(X|\Theta)] + Var(E[X|\Theta]).$$

- If Y is a $(p_1, p_2, ..., p_n)$ mixture of random variables $X_1, X_2, ..., X_n$, then the cdf of Y is

  $$F_Y(y) = p_1 F_{X_1}(y) + p_2 F_{X_2}(y) + \cdots + p_n F_{X_n}(y),$$

  where $p_i \in (0, 1), i = 1, 2, ..., n$ and $p_1 + p_2 + \cdots + p_n = 1$.

- Poisson with parameter λ:

$$Pr(N = k) = \frac{e^{-\lambda}\lambda^k}{k!}, \quad pgf : P_N(z) = e^{\lambda(z-1)}.$$

- Negative binomial distribution with parameters (r, β):

$$Pr(N = k) = \left(\begin{array}{c} k + r - 1 \\ k \end{array} \right) \left(\frac{1}{1+\beta}\right)^r \left(\frac{\beta}{1+\beta}\right)^k, \quad k = 0, 1, 2, \cdots .$$

$$pgf : P_N(z) = [1 - \beta(z-1)]^{-r}.$$

- Binomial distribution with parameters (m, q):

$$Pr(N = k) = \left(\begin{array}{c} m \\ k \end{array} \right) q^k (1-q)^{m-k}, \quad k = 0, 1, 2, \cdots, m.$$

$$pgf : \qquad P_N(z) = [1 + q(z-1)]^m.$$

- About Gamma function:

$$\begin{aligned} \Gamma(x) &= \int_0^\infty t^{x-1} e^{-t} dt, \quad x > 0, \\ \Gamma(x+1) &= x\Gamma(x), \\ \Gamma(1) &= 1. \end{aligned}$$