MA (ST) 413 Assignment 7 Solutions

1. (Problem 8.31) When $d = 500$, the distribution of N^P is zero-truncated logarithmic with $\beta = 4$. Its pgf is (see page 686)

$$P_{N^P}(z) = 1 - \frac{\ln[1-\beta(z-1)]}{\ln(1+\beta)}.$$

On the other hand, from the Pareto distribution, we have

$$v = Pr(X > 500) = \left(\frac{\theta}{500+\theta}\right)^{\alpha} = \frac{4}{9}.$$

When $d = 0$, the frequency distribution is the distribution of N^L. So we have

$$\begin{aligned} P_{N^L}(z) &= P_{N^P}(1 - v^{-1} + zv^{-1}) = P_{N^P}(2.25z - 1.25) \\ &= 1 - \frac{\ln[1 - 2.25\beta(z-1)]}{\ln(1+\beta)} \\ &= 1 - \frac{\ln[1-9(z-1)]}{\ln 5}. \end{aligned}$$

It is easy to see that

$$Pr(N^L = 0) = P_{N^L}(0) = 1 - \frac{\ln 10}{\ln 5} < 0.$$

A negative probability is not possible. Therefore, there is no appropriate model for frequency distribution when the deductible is lowered.

2. (Problem 8.33) Before the deductible, the expected number of losses is

$$E[N^L] = r\beta = 3 \cdot 5 = 15.$$

In addition, from the Weibull distribution, we have

$$v = Pr(X > 200) = S_X(200) = e^{-\left(\frac{200}{1000}\right)^{0.3}} = 0.5395.$$

Therefore, we have

$$E[N^P] = E[I]E[N^L] = vE[N^L] = 0.5395 \cdot 15 = 8.0925.$$

3. (Problem 9.6) Let X_1 be the charge for room, and X_2 is other charge. The cost for the insurance company is

$$X = X_1 + 0.8X_2.$$

Therefore, we have

$$\begin{aligned} E[X] &= E[X_1] + 0.8E[X_2] = 1000 + 0.8 \cdot 500 = 1400; \\ Var(X) &= Var(X_1) + 0.8^2 Var(X_2) + 2 \cdot 0.8 \cdot Cov(X_1, X_2) \\ &= 500^2 + 0.64(300)^2 + 2(0.8)(100000) \\ &= 467,600. \end{aligned}$$

In addition, we have $E[N] = Var(N) = 4$. So we can get

$$\begin{aligned} E[S] &= E[N]E[X] = 4(1400) = 5600; \\ Var(S) &= E[N]Var(X) + (E[X])^2 Var(N) \\ &= 4(467600) + (1400)^2 \cdot 4 = 9,710,400. \end{aligned}$$

4. (Problem 9.8) This is a mixed distribution. Given the parameter λ, the frequency N has a Poisson distribution. On the other hand, λ has the distribution as given in Table 9.4. Then we have

$$\begin{aligned} E[\lambda] &= 0.25 \cdot 5 + 0.25 \cdot 3 + 0.5 \cdot 2 = 3, \\ E[\lambda^2] &= 0.25 \cdot 5^2 + 0.25 \cdot 3^2 + 0.5 \cdot 2^2 = 10.5, \\ Var(\lambda) &= E[\lambda^2] - (E[N])^2 = 1.5. \end{aligned}$$

Therefore, we can get

$$\begin{aligned} Var(N) &= E[Var(N|\lambda)] + Var(E[N|\lambda]) \\ &= E[\lambda] + Var(\lambda) = 3 + 1.5 = 4.5. \end{aligned}$$

5. (Problem 9.9) Using the convolution process, we can get

k	$f_{X_1}(k)$	$f_{X_2}(k)$	$f_{X_1+X_2}(k)$	$f_{X_3}(k)$	$f_{X_1+X_2+X_3}(k)$
0	0.9	0.5	0.45	0.25	0.1125
1	0.1	0.3	0.32	0.25	0.1925
2	0	0.2	0.21	0.25	0.2450
3	0	0	0.02	0.25	0.2500
4	0	0	0	0	0.1375
5	0	0	0	0	0.0575
6	0	0	0	0	0.0050