# MA (ST) 413 Assignment 6
# Solutions

1. (8.14) It is not hard to get that

$$E[X \wedge 150,000] = 30,000[1 - e^{-0.00001(150,000)}] = 23,306.10.$$

After 10% inflation the expected cost is

$$1.1E\left[X \wedge \frac{150,000}{1.1}\right] = 33,000[1 - e^{-0.00001(150,000/1.1)}] = 24,560.94.$$

There is an increase of 5.38%.

2. (8.18) The quantity we seek is

$$\frac{1.1(E[X \wedge 22/1.1] - E[X \wedge 11/1.1])}{E[X \wedge 22] - E[X \wedge 11]} = \frac{1.1(17.25 - 10)}{18.1 - 10.95} = 1.115.$$

3. (8.19) This question is for per-loss variable. The expected value is

$$E[X] - E[X \wedge 100] = 1000 - 1000(1 - e^{-\frac{100}{1000}}) = 904.84.$$

To obtain the second moment, we need

$$\begin{aligned} E[(X \wedge 100)^2] &= \int_0^{100} x^2 0.001 e^{-0.001x} dx + 100^2 e^{-\frac{100}{1000}} \\ &= 9357.68. \end{aligned}$$

The second moment is

$$\begin{aligned} & E[X^2] - E[(X \wedge 100)^2] - 200E[X] + 200E[X \wedge 100] \\ = \; & 2(1000)^2 - 9357.68 - 200(1000) + 200(95.16) \\ = \; & 1,809,674.32. \end{aligned}$$

The variance is

$$1,809,674.32 - 904.84^2 = 990,938.89.$$

4. (8.22) For this uniform distribution,

$$\begin{aligned} E[X \wedge u] &= \int_0^u 0.0002x dx + \int_u^{50000} 0.00002u dx \\ &= 0.00001u^2 + 0.00002u(50000 - u) \\ &= u - 0.00001u^2. \end{aligned}$$

The expected payment per payment is

$$\frac{E[X \wedge 25000] - E[X \wedge 5000]}{1 - F_X(5000)} = \frac{18750 - 4750}{1 - \frac{5000}{50000}} = 15556.$$

5. (8.23) This is a combination of franchaise deductible and policy limit. From the definition of the policy and using the lognormal distribution, the expected cost per loss is

$$\begin{aligned}
&\int_{50,000}^{1000,000} x f_X(x)dx + 100,000[1 - F_X(100,000)] \\
= &\int_0^{1000,000} x f_X(x)dx - \int_0^{50,000} x f_X(x)dx + 100,000[1 - F_X(100,000)] \\
= &\ e^{10.5}\Phi\left(\frac{\ln 100,000 - 10 - 1}{1}\right) - e^{10.5}\Phi\left(\frac{\ln 50,000 - 10 - 1}{1}\right) \\
&+ 100,000\left[1 - \Phi\left(\frac{\ln 100,000 - 10}{1}\right)\right] \\
= &\ e^{10.5}\Phi(0.513) - e^{10.5}\Phi\left(-0.180\right) + 100,000\left[1 - \Phi\left(1.513\right)\right] \\
= &\ e^{10.5}(0.6959) - e^{10.5}(0.4285) + 100,000(0.0652) \\
= &\ 16,231.
\end{aligned}$$

6. (8.27) The expected payment is $\lambda = 3$. With the coinsurance, the expected payment is $3\alpha$. For the deductible,

$$E[X \wedge 2] = 0 \cdot e^{-3} + 1 \cdot 3e^{-3} + 2(1 - e^{-3} - 3e^{-3}) = 1.751065.$$

The expected cost is $3 - 1.751065 = 1.248935$. Then, we can get $\alpha = 1.248935/3 = 0.4163$.