MA (ST) 413 Assignment 5
Solutions

1. (8.1) For the per-payment variable,

$$\begin{aligned} f_{Y^P}(y) &= \frac{0.000003e^{-0.00001(y+5000)}}{0.3e^{-0.00001(5000)}} = 0.00001e^{-0.00001y}, \\ F_{Y^P}(y) &= 1 - e^{-0.00001y}, \quad y > 0. \end{aligned}$$

For the per-loss (left censored and shifted) variable,

$$\begin{aligned} f_{Y^L}(y) &= \begin{cases} 1 - 0.3e^{-.0.05} = 0.71463, & y = 0 \\ 0.000003e^{-0.00001(y+5000)}, & y > 0. \end{cases} \\ F_{Y^L}(y) &= \begin{cases} 0.71463, & y = 0 \\ 1 - 0.3e^{-0.00001(y+5000)}, & y > 0. \end{cases} \end{aligned}$$

2. (8.2) For the per-payment variable,

$$\begin{aligned} f_{Y^P}(y) &= \frac{0.000003e^{-0.00001y}}{0.3e^{-0.00001(5000)}} = 0.00001e^{-0.00001(y-5000)}, \\ F_{Y^P}(y) &= 1 - e^{-0.00001(y-5000)}, \quad y > 5000. \end{aligned}$$

For the per-loss (left censored and shifted) variable,

$$\begin{aligned} f_{Y^L}(y) &= \begin{cases} 1 - 0.3e^{-.0.05} = 0.71463, & y = 0 \\ 0.000003e^{-0.00001y}, & y > 5000. \end{cases} \\ F_{Y^L}(y) &= \begin{cases} 0.71463, & 0 \le y \le 5000 \\ 1 - 0.3e^{-0.00001y}, & y > 5000. \end{cases} \end{aligned}$$

3. (8.4) For risk 1, we have

$$\begin{aligned} E[X] - E[X \wedge d] &= \frac{\theta}{\alpha - 1} - \frac{\theta}{\alpha - 1}\left[1 - \left(\frac{\theta}{\theta + k}\right)^{\alpha - 1}\right] \\ &= \frac{\theta^\alpha}{(\alpha - 1)(\theta + k)^{\alpha - 1}} \end{aligned}$$

Then the ratio of risk 2 to risk 1 is

$$\frac{\frac{\theta^{0.8\alpha}}{(0.8\alpha - 1)(\theta + k)^{0.8\alpha - 1}}}{\frac{\theta^\alpha}{(\alpha - 1)(\theta + k)^{\alpha - 1}}} = \frac{(\theta + k)^{0.2\alpha}(\alpha - 1)}{\theta^{0.2\alpha}(0.8\alpha - 1)},$$

which goes to ∞ as $k \to \infty$.

4. (8.9) The loss elimination ratio is

$$\begin{aligned}\frac{E[X\wedge 2k]}{E[X]} &= \frac{\frac{\theta}{\alpha-1}\left[1-\left(\frac{\theta}{2k+\theta}\right)^{\alpha-1}\right]}{\frac{\theta}{\alpha-1}} \\ &= \frac{\frac{\theta}{2-1}\left[1-\left(\frac{\theta}{2k+\theta}\right)^{2-1}\right]}{\frac{\theta}{2-1}} \\ &= 1-\frac{\theta}{2k+\theta}\end{aligned}$$

Priori to inflation, $\theta = k$, and we have

$$\frac{E[X\wedge 2k]}{E[X]} = 1-\frac{k}{2k+k} = \frac{2}{3}.$$

After the 100% inflation, the new loss distribution is a Pareto distribution with $\alpha = 2$ and $\theta = 2k$, because θ is a scale parameter. Therefore, after the inflation, we have

$$\frac{E[X\wedge 2k]}{E[X]} = 1-\frac{2k}{2k+2k} = \frac{1}{2}.$$

5. (8.10) The original loss elimination ratio is

$$\frac{E[X\wedge 500]}{E[X]} = \frac{1000(1-e^{-\frac{500}{1000}})}{1000} = 0.39347.$$

Choose a new deductible d such that it is doubled:

$$0.78694 = \frac{1000(1-e^{-\frac{d}{1000}})}{1000} = 1-e^{-\frac{d}{1000}}.$$

The solution is $d = 1546$.

6. (8.13) The desired quantity is the expected value of a right truncated variable. It is

$$\begin{aligned}\frac{\int_0^{1000} x f_X(x)dx}{F_X(1000)} &= \frac{E[X\wedge 1000]-1000(1-F_X(1000)}{F_X(1000)} \\ &= \frac{E[X\wedge 1000]-400}{.6}.\end{aligned}$$

On the other hand, from the loss elimination ratio, we have

$$0.3 = \frac{E[X\wedge 1000]}{E[X]} = \frac{E[X\wedge 1000]}{2000}.$$

So we can get $E[X\wedge 1000] = 600$. The final answer is

$$\frac{E[X\wedge 1000]-400}{.6} = \frac{200}{.6} = 333.$$