MA(ST) 413 Assignment 1

1. The probability density function (pdf) of a random variable X with the uniform distribution on the interval $[0, 2]$ is given by

$$f(x) = \begin{cases} \frac{1}{2}, & \text{if } 0 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

   (a.) Find out the cumulative distribution function (cdf) $F(x)$, the survival function $S(x)$, the hazard rate function $h(x)$ of this random variable X.
   (b.) Compute the mean and the variance of X.
   (c.) Compute the $80th$ percentile $\pi_{0.8}$.

2. A random variable X follows an exponential distribution with the cdf

$$F(x) = \begin{cases} 1 - e^{-\lambda x}, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0, \end{cases}$$

   where $\lambda > 0$ is a constant.
   (a.) Find out the probability density function (pdf) $f(x)$, the survival function $S(x)$, the hazard rate function $h(x)$ of this random variable X.
   (b.) Compute the mean and the variance of X.
   (c.) Compute the $40th$ percentile $\pi_{0.4}$.

3. An insurance policy in an electrical device pays a benefit of 3000 if the device fails the first year. The amount of benefit decreases by 1000 each successive year until it reaches 0. If the device has not failed by the beginning of any given year, the probability of failure during that year is 0.4. what is the expected benefit under this policy?

4. The probability generating function (pgf) of a counting random variable X is $P(z) = [1 + 0.4(z - 1)]^4$. Find out its probability function, mean and variance.

5. **Loss Models**, page 29, problem 3.19.

6. **Loss Models**, page 34, problem 3.24.