
Index

- A**
- a posteriori analysis 145
 - accuracy . *see* computational methods
 - first order 264, 265, 268, 312
 - order of 265, 309, 312, 322
 - for quadrature formulae . . 93–95
 - second order 266, 269, 448
 - active control 487, 503
 - general solution for . . . 503, 504
 - active control source *see* active control
 - active shielding 487, 503, 506
 - amplification factor 206, 351
 - for the first order upwind scheme 342
 - for the Lax-Wendroff scheme 343
 - amplification matrix 366, 388
 - antiderivative 91
 - approximate
 - computations 1
 - evaluation
 - of a function 13
 - of integrals 23
 - solution 2, 5, 9, 10
 - value 8
 - of a function 23
 - approximation
 - accuracy of 5
 - best
 - by polynomials 84
 - in linear space 88
 - in the sense of C 88
 - in the sense of L_2 88
 - by difference scheme . . 259, 260, 265, 309
 - order of 309
 - by rounding 11
 - discrete 5
 - error 5, 8, 11
 - of differential operator by difference operator 265, 333
 - of functions 9
 - order of 265, 309
 - Arnoldi process 200, 201
 - instability of 200
 - stabilized version of 200
 - termination of 200, 203
 - premature 200, 202, 203
 - artificial boundary 486, 501, 505
 - artificial boundary condition 485, 486, 501, 506
 - non-reflecting 505
 - radiation 505
 - artificial dissipation *see* viscosity, artificial
 - artificial viscosity *see* viscosity, artificial
 - auxiliary difference problem 488, 490, 498, 499
- B**
- bandwidth 147
 - basis
 - in a linear space 63, 123, 125
 - orthonormal 64, 130, 136, 164, 166, 200
 - Bernstein example 34, 87
 - bisection . *see* method, of rootfinding, bisection
 - boundary equation with projection 492, 498–501
 - Calderon's 473, 483, 492

C

- Cauchy type integral 483, 497
 CFL condition 318, 319
 CFL number 318
 characteristics *.see* equation, Burgers,
 characteristics of
 intersection of 431, 438
 method of *see* method, of
 characteristics
 Chebyshev polynomials 76
 orthogonality of 103
 roots of 77
 stability of computation of . . . 79
 Cholesky factorization *see*
 factorization, Cholesky
 chord method *see* method, of
 rootfinding, chord
 component 121, 124
 computational methods . . . 3, 12–18
 characteristics of 3
 accuracy 3, 13
 convergence 18
 operation count 3, 14
 stability 14
 storage requirements 3
 comparison of 13–18
 by convergence 18
 by efficacy 14
 by stability 14–15
 in accuracy 13
 parallelization of 3, 18
 condition at the discontinuity . . . *see*
 discontinuity, condition at
 condition number 133
 as a characteristic of $Ax = f$ 137
 geometric meaning of 135
 of a linear operator 134
 of a linear system 134
 of a matrix 134
 of a matrix with diagonal domi-
 nance 139
 of a self-adjoint operator . . . 136
 of rootfinding problem 232
 conditioning 3, 6, 16, 17
 ill conditioned problems . . . *see*
 problem, ill conditioned
 of interpolating polynomial . 32–
 33
 of linear systems 133
 of rootfinding problem 232
 well conditioned problems *.see*
 problem, well conditioned
 conservation law 427, 428
 integral form of 427, 428
 consistency
 of approximating operator equa-
 tion 321
 of finite-difference scheme *.see*
 finite-difference scheme,
 consistency of
 contact discontinuity 427
 contraction 238
 coefficient 238
 sufficient condition for 240
 contraction mapping principle . . . 239
 convergence
 of finite element method *.see* fi-
 nite elements, convergence
 of
 of finite-difference scheme *.see*
 finite-difference scheme,
 convergence of
 of finite-difference solution *.see*
 finite-difference solution,
 convergence of
 of Fourier series *see* series,
 Fourier, convergence of
 of interpolating polynomials 34,
 83, 86
 on Chebyshev grid . 77, 80, 87
 of iterative method
 a necessary and sufficient con-
 dition for 178,
 182
 a sufficient condition for . 175
 first order linear stationary
 175, 178
 fixed point . . . *see* fixed point
 iteration, convergence of

- Gauss-Seidel 177
 - Jacobi 176
 - Richardson 182
 - with a self-adjoint iteration matrix 181
 - of local splines 47
 - with derivatives 47
 - of Newton's method . . . 236, 292
 - quadratic 236, 243–245
 - of nonlocal splines 53
 - with derivatives 53
 - of piecewise polynomial interpolation 41
 - for derivatives 41
 - of solutions to approximate operator equations 321
 - of the chord method 235
 - of the method of bisection . . 234
 - of the secant method 235
 - of trigonometric interpolation 61, 71
 - spectral
 - of adaptive finite elements 469
 - of Fourier method for a differential equation 306
 - of Gaussian quadratures . 106, 107
 - of Newton-Cotes quadratures for periodic functions . . . 97
 - of trigonometric interpolation 73
 - convex domain 241
 - convolution 476
 - Courant number 318
 - Courant, Friedrichs, and Lewy condition 318
 - Cramer's rule 119
- D**
- diagonal dominance *see* matrix, with diagonal dominance
 - difference
 - central 262, 265, 447
 - divided 26–27, 45
 - forward 262
 - quotient *see* quotient, difference
 - discontinuity 431
 - condition at 432
 - formation of 431, 433
 - of the first kind . . . 431, 433, 454
 - tracking of 439
 - trajectory of 432
 - discretization 3, 4
 - comparison of methods 5
 - of a boundary value problem . . 5
 - of a function 4
 - by table of values 4
 - of derivatives 4
 - dispersion 341
 - of the first order upwind scheme 342
 - of the Lax-Wendroff scheme 343, 344
 - dispersion relation 341
 - for the first order upwind scheme 342
 - for the Lax-Wendroff scheme 343
 - dispersionless propagation of waves 342
 - domain of dependence
 - continuous 318
 - discrete 318
 - dot product 63
 - double precision *see* precision, single
 - Duhamel principle 365
- E**
- elimination
 - Gaussian 140
 - computational complexity of 145, 152
 - standard 141
 - sufficient condition of applicability 142
 - with pivoting 154
 - with pivoting, complexity of 155
 - tri-diagonal . . 51, 145, 255, 276, 291

- computational complexity of 146
 - cyclic 52, 148
 - cyclic, complexity of 149
- elliptic
 - boundary value problem *see* problem, boundary value, elliptic
 - equation *see* equation, elliptic
 - operator *see* operator, elliptic
- energy estimate 402
 - for a constant-coefficient equation 403
 - for a finite-difference scheme *see* finite-difference scheme, energy estimate for
 - for a variable-coefficient equation 404
 - with inhomogeneous boundary condition 405
- entropy condition 436
- equality
 - Parseval 167, 363
- equation
 - boundary, with projection *see* boundary equation with projection
 - Burgers 428
 - characteristics of 430
 - divergence form of 441
 - Calderon *see* boundary equation with projection, Calderon's
 - d'Alembert 348, 358
 - difference 78
 - elliptic 121, 445–470
 - with variable coefficients 187, 445
 - Fredholm integral of the second kind 477
 - heat 325, 330, 356
 - homogeneous 346
 - nonlinear 370
 - two-dimensional 358
 - Helmholtz 460, 478, 481
 - Lagrange-Euler 457
 - Laplace 475, 485, 498
 - linear algebraic 120
 - Poisson 121, 205, 332, 445, 446, 454, 476, 481, 484
 - system of
 - acoustics 326, 360
 - Lame 481
 - linearized Euler 481
 - Maxwell 481
 - Stokes 479, 481
 - wave 348, 358
- error 3, 8
 - asymptotic behavior for interpolation 41
 - estimate 9, 38, 39
 - for algebraic interpolating polynomials 86
 - for Chebyshev iteration . . 195
 - for conjugate gradients . . 196
 - for first order linear stationary iteration 176, 180
 - for fixed point iteration . . 238
 - for Gaussian quadrature . . 105
 - for local splines 46
 - for nonlocal splines 53
 - for quadrature formulae . . 92
 - for Richardson iteration . . 185
 - for Simpson's formula . 98, 99
 - for spectral method 305
 - for the method of bisection 234
 - for trapezoidal rule 94
 - for unavoidable error . . 40–42
 - of algebraic interpolation
 - on Chebyshev grid 77, 78
 - of finite-difference solution . *see* finite-difference solution, error of
 - of iteration 175, 179, 182
 - of piecewise polynomial interpolation 35–38
 - for derivatives 39
 - multi-dimensional 58
 - of Taylor formula *see* remainder, of Taylor formula

- of the computational method . 8, 10
 - of trapezoidal rule 278
 - of trigonometric interpolation 61, 68
 - order of 322
 - phase 341–343
 - minimization of 343
 - relative 15, 281
 - round-off . . . 8, 10–11, 278–279
 - truncation . . . 259, 261, 309, 448
 - magnitude of 262–264
 - unavoidable 8–10, 42, 87
 - due to uncertainty in the data 8–10
- F**
- factorization
 - LU** 151
 - QR** 217
 - relation to Cholesky 218
 - Cholesky 153
 - complexity of 153
 - family of matrices 366, 388
 - stability of 366, 388
 - family of operators 389, 391
 - behavior of the powers R_h^p . . 394
 - spectrum of 385, 391, 411
 - computation using Babenko-Gelfand procedure 397–402
 - uniform boundedness of powers 387
 - fan 435, 443
 - fast Fourier transform (FFT) 169, 171
 - Favard constants 85
 - finite elements
 - adaptive 468
 - basis for 465
 - convergence of 466–469
 - optimal 467
 - spectral 469
 - method of *see* method, finite element
 - piecewise linear 465
 - finite-difference scheme . . . 251, 255, 308
 - A-stable 281
 - conditionally 282
 - unconditionally 282
 - accuracy of . *see* accuracy, order of
 - backward Euler 271
 - canonical form of 386
 - central difference 333
 - conservative 441, 442
 - consistency of . . . 259, 261, 309
 - consistent 309, 310
 - convergence of . . 257, 259, 273, 308
 - order of 259, 308
 - convergent 256, 277, 309
 - Crank-Nicolson
 - for ordinary differential equations 270
 - for partial differential equations 373
 - dispersion of *see* dispersion
 - dissipation of 372
 - artificial *see* viscosity, artificial
 - divergent 271
 - energy estimate for
 - constant coefficients 407
 - inhomogeneous boundary conditions 409
 - variable coefficients 408
 - evolution-type 385
 - explicit 270, 285, 311
 - explicit central 354
 - first order downwind 329
 - instability of 353
 - first order upwind 329
 - Babenko-Gelfand stability of 382–384
 - dispersion of *see* dispersion, of the first order upwind scheme
 - dissipation of 373

- for initial boundary value problem 382, 385
 - von Neumann stability of 351
- for an integral equation 277
 - stability of 278
- for systems 286
- for the heat equation 330
 - Babenko-Gelfand stability of 380–381
 - explicit 331, 356, 423
 - implicit . . . 147, 331, 357, 426
 - on a finite interval 377
 - two-dimensional 358
 - weighted 367
 - with variable coefficients . 369
- for the Laplace equation
 - stability of 447, 448
- for the Poisson equation . . . 122
- forward Euler 269, 284, 344
 - convergence of 276
 - for systems 287
 - stability of 276
- Godunov 442
- Heun 270
- implicit 270
- implicit central 368
- implicit downwind 368
- implicit upwind 368
- instability of 320
- Lax-Friedrichs 345
 - von Neumann stability of 355
- Lax-Wendroff . . . 339, 343, 345, 353
 - Babenko-Gelfand stability of 418
 - dispersion of *see* dispersion, of the Lax-Wendroff scheme
 - dissipation of 373
 - for acoustics system 361
 - for inhomogeneous equation 347
 - von Neumann stability of 354
- leap-frog 329, 345
 - Kreiss stability of . . 410–418
 - von Neumann stability of 356
- linear 282
 - multi-step 282
 - single-step 282
 - stability of 283
- MacCormack 345
- monotone 347
- non-dissipative 372
- order of accuracy . *see* accuracy, order of
- predictor-corrector . . . 270, 344, 444
- Runge-Kutta 284, 344
 - for second order equation 288
 - fourth order 285
 - second order 285
- saturation of 293–300
- spectrum of
 - for Cauchy problem . 351–361
 - for initial boundary value problem 379–385
- stability constants 280–281
- stability of . . 260, 272, 312, 448
 - “almost” sufficient condition in terms of the spectrum of a family of operators . . 395
 - as energy estimate 409
 - as uniform boundedness of R_h^p 387
- Babenko-Gelfand criterion 379
 - for initial boundary value problems 377–418
 - for linear equations . 273, 312
 - for nonlinear equations . . 273, 312
- Godunov-Ryaben’kii
 - necessary condition . . . 391
- in C 362
- in l_2 362
- Kreiss necessary and sufficient condition 416
- spectral condition for Cauchy problems 351
- spectral condition for initial boundary value problems

- 389
 - spectral theory 283, 351
 - sufficient conditions in C . 362
 - unconditional 422, 426
 - von Neumann analysis of 349
 - von Neumann condition of 352
 - with respect to initial data 350
 - stable 309
 - stencil of . . . 122, 293, 324, 330, 332, 425
 - five-node 333, 484
 - four-node . 337, 339, 341, 422
 - three-node 334, 341
 - finite-difference solution
 - convergence of 258, 308
 - rate of . . . 259, 273, 308, 314, 322
 - error of 259, 308
 - characterization by grid dimension 322
 - characterization by grid size 322
 - order of 259, 308
 - to an integral equation
 - convergence of 278
 - fixed point 237, 239
 - fixed point iteration 237–242
 - convergence of 237
 - sufficient condition for . . . 239
 - flux 441
 - evaluation of 443
 - by solving a Riemann problem 444
 - FOM 200
 - formula
 - first Green's 455
 - Green's 479
 - Hopf 436
 - Newton-Cotes 97
 - Newton-Leibniz 91
 - quadrature 23, 91, 92
 - error of *see* error, estimate, for quadrature formulae
 - Gaussian *see* Gaussian quadrature
 - second Green's 455
 - Simpson's 93, 98
 - saturation of 101
 - Taylor . . . 10, 14, 100, 232, 262, 266, 285, 327, 447
 - error of *see* remainder, of Taylor formula
 - remainder of . *see* remainder, of Taylor formula
 - frequency 341
 - function 23
 - absolutely integrable 87
 - indicator 486
 - Lipshitz-continuous . 84, 87, 107
 - piecewise analytic 497
 - quadratic 157
 - minimization of 158
 - reconstruction of 23
 - from table of Taylor coefficients 23
 - from table of values . . . 23, 40
 - table of values of 25, 42, 68
 - fundamental solution . . . *see* solution, fundamental
- G**
- Gauss nodes *see* grid, Chebyshev-Gauss
 - Gauss-Lobatto nodes *see* grid, Chebyshev-Gauss-Lobatto
 - Gaussian quadrature 102, 105
 - error of . . *see* error, estimate, for Gaussian quadrature
 - exactness for polynomials of degree $\leq 2n + 1$ 107
 - no saturation of 102, 105
 - optimality of 107
 - generalized eigenvalue 416
 - GMRES 200, 217
 - instability of 204
 - Gram-Schmidt orthogonalization 200, 218
 - stabilized 219
 - Green's function

free-space *see* solution,
fundamental

grid 5, 9
adaptation of 256
Cartesian 57, 122, 205, 310, 328,
447, 484
Chebyshev . . . 33, 62, 77, 79, 87
Chebyshev-Gauss . 77, 102, 103
Chebyshev-Gauss-Lobatto . 80,
102, 103
dimension of 256
equally spaced *see* grid, uniform
finite element 464
finite-difference . 251, 254, 308
for quadrature formulae 92
nonuniform 102, 255
rectangular 57, 167
refinement of 41, 47
local 53, 59
size 38, 255
structured 57
triangular 59, 464
uniform 33, 40, 57,
61, 62, 93, 122, 167, 205,
254, 257, 258, 261, 277,
284, 310, 447, 484
unstructured 57, 59, 464

grid boundary 490, 501

grid function 123, 127, 165, 251, 254,
488–504

even 66
norm of *see* norm, of a grid
function

odd 66

piecewise regular 493, 495

two-valued 493

vector 366

I

ill-posedness 18

improper integrals
computation of 108
by change of variables . . . 110
by combined analyti-
cal/numerical methods . 108

by regularization 109, 110
by splitting 108, 109
using integration by parts 108

inequality
Bessel 69
Cauchy-Schwarz 132
Friedrichs 457
Hölder 70
Jackson 84
Lebesgue 85
triangle 126

initial guess 173, 174

inner product 63, 126, 409

instability 14–15
due to the loss of significant dig-
its 16,
17
of finite-difference scheme . *see*
finite-difference scheme,
instability of
of iterative method . *see* method,
iterative, instability of

interpolating polynomial 25
algebraic 25
comparison of different forms
31
conditioning of 32–33
convergence of 34
divergence of 34
existence of 25–26
Lagrange form of 26, 31
Newton form of 27, 31
on Chebyshev grid . . . 77, 79
sensitivity to perturbations 32
uniqueness of 25–26

trigonometric 61, 62, 72
convergence of 73
for even grid functions 66, 73,
75
for odd functions . . . 301–302
for odd grid functions 66
sensitivity to perturbations 67
stability of 73

interpolation 23
algebraic 23, 25

- no saturation on Chebyshev grid 78, 80, 87
 - on Chebyshev grid 77, 79
 - multi-dimensional 57–60
 - error estimate of 58, 60
 - on structured grids 57–58
 - on unstructured grids 58–60
 - saturation of 58
 - piecewise polynomial 35–43
 - choice of degree 40–42
 - convergence of 41
 - error estimate of 35–38
 - of derivatives 38–39
 - saturation of 42
 - smooth 43
 - trigonometric 23, 61
 - convergence of 61, 71, 73
 - error estimate of 61, 68, 72
 - no saturation of 71, 73
 - of functions on an interval 75
 - of periodic functions 62
 - sensitivity to perturbations 61, 67
 - interpolation grid 33, 35, 49
 - effect on convergence 87
 - iterate 174
 - iteration 173
 - convergence of *see* convergence, of iterative method
 - error of . . . *see* error, of iteration
 - matrix 174
 - scheme 173
- J**
- Jacobi matrix *see* matrix, Jacobi
 - Jacobian *see* matrix, Jacobi
 - jump 431, 433
 - across the discontinuity 432, 433
 - trajectory of 432, 433
 - velocity of 432, 433
- K**
- Kolmogorov diameter 41, 88, 467
 - Krylov subspace 199
 - basis of 200
- iteration . *see* method, of Krylov subspaces
- L**
- Lebesgue constants 32, 81, 86
 - growth of 86
 - of algebraic interpolating polynomial 32–33, 41, 82
 - on Chebyshev grid 33, 77, 78, 80, 87
 - on uniform grid 33
 - optimal asymptotic behavior of 87
 - of trigonometric interpolating polynomial 61, 67, 72, 73
 - Legendre polynomial 224
 - linear combination 63, 65, 88
 - linear dependence 125
 - linear independence 125
 - Lipshitz-continuity *see* function, Lipshitz-continuous
 - loss of significant digits 15–17
 - LU factorization . . . *see* factorization, *LU*
- M**
- mapping
 - contraction *see* contraction
 - marching 255, 269
 - for tri-diagonal systems 146
 - in time *see* time marching
 - mathematical model(s) 1–3
 - for small oscillations 13
 - methods of analysis 1–3
 - computational 1–3
 - matrix
 - banded 146
 - block-diagonal 394
 - condition number of . *see* condition number, of a matrix
 - conjugate 128
 - diagonal 176, 192
 - full rank 215, 218
 - Gram 460

- Hermitian 128
- Jacobi 241, 244, 287
- kernel of 225
- lower triangular 151
- Moore-Penrose pseudoinverse
226
- computation of 227
- norm of . . . *see* norm, of a matrix
- normal 131, 136, 366, 390
- permutation 155
- rank deficient 225
- rectangular 214
- full rank 215
- SPD 128, 153, 216–218
- symmetric 128
- transposed 128
- tri-diagonal 145
- with diagonal dominance . 145
- upper Hessenberg 201
- upper triangular 151, 218
- with diagonal dominance . 138,
426
- estimate for condition number
139
- Gauss-Seidel iteration for 177
- Gaussian elimination for . 142
- Jacobi iteration for 176
- maximum principle
- for differential Laplace equation
450
- for finite-difference equation
 $u_t - u_x = 0$ 315
- for finite-difference heat equa-
tion 325, 423,
426
- for finite-difference Laplace
equation 449
- strengthened 452
- for Godunov scheme 443
- for monotone schemes 347
- method
- Broyden 246
- Chebyshev iterative 194
- first order 196
- second order 195
- direct . . 119, 140, 162, 164, 173
- with linear complexity . . 146
- finite element 453–470
- convergence of *see* finite
 elements, convergence of
- finite-difference *see*
 finite-difference scheme
- Galerkin 462, 465
- Gauss-Seidel 177, 188
- convergence of 177
- iterative 119, 134, 162, 173
- Arnoldi *see* FOM
- Chebyshev *see* method,
 Chebyshev iterative
- error of . *see* error, of iteration
- first order linear stationary
174, 178
- fixed point *see* fixed point
 iteration
- instability of 180
- multigrid *see* multigrid
- of conjugate gradients . . . *see*
 method, of conjugate gradi-
 ents, as an iterative method
- Picard *see* Picard iteration
- second order linear stationary
195, 196
- stability of 180
- Jacobi 176, 188
- convergence of 176
- JOR 178
- Kholodov's of constructing
 schemes 341
- Lanczos 163
- Monte Carlo 113
- Newton's 242–246, 291
- convergence of . . *see* conver-
 gence, of Newton's method
- fixed point interpretation . 243
- for scalar equations 242
- for systems 244
- geometric interpretation . 243
- modified 246
- Newton-Jacobi 246
- Newton-Krylov 246

- Newton-SOR 246
 - of boundary elements 473, 479–481
 - of boundary integral equations 473
 - of characteristics 438
 - of conjugate gradients 159, 161, 198
 - as an iterative method ... 194, 196
 - computational cost of ... 163
 - for computing exact solution 162
 - of difference potentials 473, 483–504
 - with difference approximation of boundary conditions 498
 - with spectral approximation of boundary conditions 499
 - of finite differences *see* finite-difference scheme
 - of finite elements 446
 - of full orthogonalization *see* FOM
 - of generalized minimal residuals *see* GMRES
 - of Krylov subspaces 198
 - of least squares 213
 - geometric meaning of ... 220
 - of linear programming 213
 - of rootfinding 231, 289
 - bisection 233, 289
 - chord 235
 - Newton's 236, 290
 - secant 235, 290
 - of shooting 289
 - instability of 290–291
 - of undetermined coefficients 334–340
 - over-relaxation 178
 - Jacobi 178
 - successive 178
 - projection 462
 - quasi-Newton 246
 - Richardson
 - non-stationary 174, 193
 - preconditioned 188
 - stationary 174
 - Ritz 458, 465
 - SOR 178
 - spectral
 - Chebyshev 306
 - collocation 306
 - Fourier 301–306
 - Galerkin 306
 - no saturation of 306
 - minimizing sequence 457
 - multigrid 204, 210
 - algebraic 210
 - grid-independent performance of 204
 - prolongation 208
 - restriction 207
 - V-cycle 209
 - W-cycle 209
 - multiple integrals
 - computation of 111
 - by coordinate transformation 112
 - by Monte Carlo method .. 113
 - by repeated application of quadrature formulae .. 111–112
- N**
- Newton's method *see* method, of rootfinding, Newton's, *see* method, Newton's
 - norm
 - C 126, 349, 389, 390, 421
 - l_1 126, 127
 - l_2 . 127, 362, 390, 407, 410, 421
 - l_∞ 126, 349
 - axioms of 126
 - energy 459
 - equivalence on vector spaces 176
 - Euclidean .. 127, 128, 131, 257, 362
 - first 126
 - Hermitian 127, 128

- Manhattan 126
 maximum .. 126, 127, 257, 311, 349, 448
 of a grid function 257
 of a matrix 129
 maximum column 130
 maximum row 130
 of a vector 125
 of an operator 81, 129
 induced by a vector norm 129
 on a linear space 126
 Sobolev 454
- O**
- operation count ... *see* computational methods
 operator
 adjoint 128
 condition number of *see* condition number, of a linear operator
 continuous 81
 elliptic 190
 equivalent by energy 190
 equivalent by spectrum 190
 Helmholtz 461, 478
 Laplace 191, 476
 eigenvalues of 461
 finite-difference 122, 171, 448
 linear ... 81, 121, 124, 129, 273
 matrix of 121, 124
 orthogonal 132
 unitary 132
 norm of *see* norm, of an operator
 positive definite .. 128, 168, 456
 self-adjoint 128, 164, 168
 transition 351, 386
 boundedness of powers of 387
 eigenfunctions of ... 351, 389
 eigenvalues of . 351, 379, 389
 spectrum of ... 351, 372, 383, 389
- P**
- periodization
 by cosine transformation ... 74
 by extension 74
 phase error *see* error, phase
 phase speed 341
 for the first order upwind scheme 342
 for the Lax-Wendroff scheme 343
 Picard iteration 237
 piecewise regular function ... *see* grid function, piecewise regular
 piecewise regular solution *see* solution, piecewise regular
 pivot 151
 pivoting 154
 complete 155
 partial 154
 potential
 difference 488–504
 $u^+ = \mathbf{P}^+ v_\gamma$ 489, 490
 $u^- = \mathbf{P}^- v_\gamma$ 492
 $u_N^+ = \mathbf{P}^+ v_\gamma$ 498
 Cauchy type .. 483, 493, 495, 498, 502
 with jump v_γ ... 493, 495, 502
 double-layer 476
 density of 476
 single-layer 476
 density of 476
 precision
 double 279
 single 279
 precision, double 157
 preconditioner 188
 incomplete factorization ... 192
 polynomial 192
 recursive ordering 192, 210
 spectrally equivalent 191
 preconditioning 188, 192
 for discrete elliptic boundary value problems ... 190, 191
 primitive 91
 principle of frozen coefficients .. 369
 problem

- auxiliary *see* auxiliary difference problem
 - boundary value 4, 253, 288, 291
 - Dirichlet 445
 - elliptic . . . 121, 158, 186, 190, 205, 445–470
 - exterior 485, 500
 - for the Helmholtz equation 461, 478
 - for the Laplace equation . 475
 - for the Poisson equation . 121
 - interior 485, 498
 - Neumann 445
 - of the first kind 445
 - of the second kind 445
 - of the third kind 445
 - Robin 445
 - variational formulation of 454, 457
 - Cauchy 13, 271, 289, 290
 - difference 349, 362, 369
 - difference vector 365
 - for a first order advection equation 309, 325, 326, 344
 - for a first order ordinary differential equation 253, 284
 - for a hyperbolic system . . 375
 - for a system of ordinary differential equations 287
 - for acoustics system 360
 - for the Burgers equation . 428
 - for the heat equation 325, 330, 346, 356, 358
 - for the nonlinear heat equation 370
 - for the wave equation . . . 358
 - Dirichlet 121, 205, 332
 - exterior 475
 - finite-difference 121, 165, 168, 205, 489
 - for the Helmholtz equation 461, 478
 - for the Laplace equation . 475, 485
 - for the Poisson equation . 121, 446, 454
 - interior 475
 - spectrum of 206
 - with variable coefficients 451, 469
 - ill conditioned 6–7
 - initial boundary value 348, 377–418
 - initial value . 253, 254, 261, 267, 274, 317
 - inverse source 487
 - Neumann
 - exterior 475
 - for the Helmholtz equation 478
 - for the Laplace equation . 475
 - interior 475
 - Riemann 434, 442
 - well conditioned 6–7, 180
 - well-posed 6
 - projection
 - boundary 492
 - Calderon's 473, 492
 - of continuous solution on the grid 256, 272, 308
 - orthogonal 220
 - independence of basis . . . 221
 - on a subspace 220
- Q**
- quadrature formulae *see* formula, quadrature
 - quotient
 - difference 27, 254, 261, 269
 - Rayleigh-Ritz *see* Rayleigh-Ritz quotient
- R**
- rarefaction wave 434
 - Rayleigh-Ritz quotient 190
 - remainder
 - of Taylor formula
 - Lagrange form . 262, 266, 447
 - residual 160, 174

of exact solution . 259, 261, 264, 309
 residue 180, 397
 resonance of the interior domain 478, 479
 Riemann invariants 420
 rootfinding
 method of *see* method, of rootfinding
 Runge example 34

S

saturation by smoothness 41, 42
 of finite-difference schemes . *see* finite-difference scheme, saturation of
 of piecewise polynomial interpolation *see* interpolation, piecewise polynomial, saturation of
 of Simpson's formula . . . *see* formula, Simpson's, saturation of
 of splines
 local *see* splines, local, saturation of
 nonlocal *see* splines, nonlocal, saturation of
 of trapezoidal rule *see* trapezoidal rule, saturation of
 saturation on a given stencil 293
 scalar product 126
 axioms of 126
 scaling 192
 scheme
 difference . . *see* finite-difference scheme
 finite-difference *see* finite-difference scheme
 secant method *see* method, of rootfinding, secant
 sensitivity to perturbations
 of interpolating polynomial algebraic 32, 35
 trigonometric 61, 67

of solution 6
 finite-difference 260, 273, 314, 319, 323
 to a linear system . . . 133, 137

series

Fourier 68, 88, 95
 coefficients of 69, 95
 convergence of . . 69, 303, 305
 discrete 167
 finite 164, 167
 finite, two-dimensional . . 167
 in the complex form 170
 partial sum of 69, 88, 95, 303, 305
 remainder of . 69, 88, 95, 303, 305
 Laurent 179
 power 23
 Taylor 13, 84
 partial sum of 13, 84

shock 323, 427, 434, 443
 rarefaction 436
 trajectory of 439

similarity of matrices 226
 Simpson's formula *see* formula, Simpson's

single precision . *see* precision, single
 singular value 226
 singular value decomposition . . . 225
 singular vector
 left 226
 right 226

solution

approximate 256, 308
 classical 433
 continuous 427
 discontinuous 427, 433
 exact 254, 258, 308
 on the grid 254, 258, 308
 finite-difference *see* finite-difference solution
 fundamental
 of the Helmholtz operator 478
 of the Laplace operator . . 476
 generalized 212, 215, 433

- computation by means of QR
 - 218
- existence and uniqueness of
 - 215
- for an operator equation.. 221
- in the sense of l_1 213
- in the sense of the least squares 213
- minimum norm 228
- of a conservation law 433
- of an overdetermined linear system 202, 212, 215
- piecewise regular. 493, 495, 502
- plane wave..... 341
- weak..... 212, 215, 433
 - computation by means of QR
 - 218
 - existence and uniqueness of
 - 215
 - for an operator equation.. 221
 - in the sense of l_1 213
 - in the sense of the least squares..... 202, 213
 - minimum norm 228
 - of a conservation law 433
 - of an overdetermined linear system 202, 212, 215
- Sommerfeld radiation boundary conditions 478
- space
 - $C[a, b]$ 81, 88
 - C^1 236, 298
 - C^2 455–462
 - C^∞ 298
 - $L_1[a, b]$ 87
 - $L_2[a, b]$ 88
 - \mathbb{C}^n 125
 - \mathbb{R}^n 125
 - l_2 362
 - affine..... 199
 - Banach 320
 - complete 239, 454
 - Euclidean..... 127
 - Hilbert 88
 - linear 63, 121, 125, 264
 - addition of elements 125
 - basis of .. *see* basis, in a linear space
 - complex 125
 - dimension of ... 63, 121, 125, 264
 - elements of 121
 - multiplication of elements by scalars 125
 - normed 126, 257, 264
 - real 125
 - Sobolev W_2^1 (or H^1) 454
 - unitary 127
 - vector 125
- spectral radius ... 130, 169, 178, 181
- spectrum
 - boundaries of 186, 194
 - sharp 194
 - of a family of operators *see* family of operators, spectrum of
 - of a matrix 130
 - boundaries of 186
 - of an operator 130, 393
 - of finite-difference problem *see* finite-difference scheme, spectrum of
 - of transition operator ... *see* operator, transition, spectrum of
- splines
 - local 43–48
 - convergence of 47
 - error estimate for 46
 - optimality of 47
 - saturation of 48, 53
 - multi-dimensional 58
 - nonlocal 43, 48–53
 - convergence of 53
 - error estimate for 53
 - optimality of 53
 - saturation of 53
 - Schoenberg's cubic 49
 - stability ... 3, 14–15, 73, 79, 80, 147

of approximating operator equation 321
 of families of matrices *see*
 family of matrices, stability
 of
 of finite-difference scheme . . . *see*
 finite-difference scheme,
 stability of
 stencil . . . *see* finite-difference scheme,
 stencil of
 subspace 199
 Krylov *see* Krylov subspace
 summation by parts 407, 409
 SVD *see* singular value
 decomposition
 system
 Cauchy-Riemann 497
 linear 117, 119
 canonical form of . . 120, 123,
 124, 140, 214
 coefficients of 119
 condition number of *see*
 condition number, of a lin-
 ear system
 conditioning of *see* condition-
 ing, of linear systems
 consistent 120
 homogeneous 120
 matrix form of 120
 matrix of 120, 214
 of high dimension 123
 operator form of . . . 121, 123,
 124
 overdetermined 202, 211, 214
 right-hand side of 119
 of linear algebraic equations *see*
 system, linear
 of scalar equations 117

T

theorem
 Bernstein 87
 Cauchy (convergence of funda-
 mental sequences) 239

convergence of consistent and
 stable schemes . . . 273, 314
 existence and uniqueness
 of algebraic interpolating
 polynomial 25
 of local spline 44
 of trigonometric interpolating
 polynomial 62, 72
 Faber-Bernstein 86
 fundamental, of calculus 91
 Godunov 347
 Godunov-Ryaben'kii 391
 Jackson inequality *see*
 inequality, Jackson
 Kantorovich 321
 Kreiss
 matrix 366
 on stability of dissipative
 schemes 376
 Lagrange (first mean value). 240
 Lax equivalence 314, 323
 Lebesgue
 inequality *see* inequality,
 Lebesgue
 on integrability of the deriva-
 tive of an absolutely contin-
 uous function 87
 maximum principle 449
 necessary and sufficient condi-
 tion for stability in l_2 . . 363
 Newton form of interpolating
 polynomial 29
 Riesz-Fischer 363
 Rolle (mean value) . . . 30, 36, 39
 stability of linear schemes for
 ordinary differential equa-
 tions 283
 Sturm 231
 Weierstrass
 on uniform approximation by
 polynomials 84
 on uniformly converging se-
 ries of holomorphic func-
 tions 179

theory

- GKS 410
 of approximation 81, 467
 Thomas algorithm 147
 time marching 311, 331
 trace
 of continuous function on the
 grid 265
 of continuous right-hand side on
 the grid 308
 of continuous solution on the
 grid 254, 256, 308, 310
 transformation
 Givens 220
 Householder 220, 227
 trapezoidal rule 93, 277
 error of . . . *see* error, estimate, for
 trapezoidal rule, 278
 no saturation for periodic func-
 tions 95,
 97
 saturation of 94
 spectral convergence for peri-
 odic functions 97
 truncation error . *see* error, truncation
- V**
- vector 125
 components of 125
 norm of . . . *see* norm, of a vector
 vectors
 A-conjugate 160
 A-orthogonal 160
 viscosity 436
 artificial 376, 437
 von Neumann stability analysis . . *see*
 finite-difference scheme,
 stability of, von Neumann
 analysis of
- W**
- wave
 incoming 505
 outgoing 505
 rarefaction 435
 shock 434
 wavelength 342
 wavenumber 341, 460, 478
 well-posedness 6, 314, 405