From the definition of $w(\lambda)$ it is easy to see that $-u^p$ is the residue of the vector-function $\lambda^{p-1}w(\lambda)$ at infinity:

$$u^p = \frac{1}{2\pi i} \oint_{|\lambda| = \gamma} \lambda^{p-1}w(\lambda)d\lambda = -\frac{1}{2\pi i} \oint_{|\lambda| = \gamma} \lambda^{p}(R - \lambda I)^{-1}u^0d\lambda.$$  

As $u^p = R^p u^0$, the last equality is equivalent to (10.158).

Altogether, we have seen that the question of stability for evolution finite-difference schemes on finite intervals reduces to studying the spectra of the families of the corresponding transition operators $\{R_h\}$. More precisely, we need to find out whether the spectrum for a given family of operators $\{R_h\}$ belongs to the unit disk $|\lambda| \leq 1$. If it does, then the scheme is either stable or, in the worst case scenario, it may only develop a mild instability.

Let us now show how we can actually calculate the spectrum of a family of operators. To demonstrate the approach, we will exploit the previously introduced example (10.142a), (10.142b). It turns out that the algorithm for computing the spectrum of the family of operators $\{R_h\}$ coincides with the Babenko-Gelfand procedure described in Section 10.5.1. Namely, we need to introduce three auxiliary operators: $\overrightarrow{R}, \overrightarrow{R},$ and $\overrightarrow{R}$. The operator $\overrightarrow{R}, v = \overrightarrow{R} u$, is defined on the linear space of bounded grid functions $u = \{u_0, u_1, u_2, \ldots\}$ according to the formula:

$$v_m = (1 - r)u_m + ru_{m+1}, \quad m = 0, \pm 1, \pm 2, \ldots,$$  

which is obtained from (10.142a), (10.142b) by removing both boundaries. The operator $\overrightarrow{R}$ is defined on the linear space of functions $u = \{u_0, u_1, \ldots, u_m, \ldots\}$ that vanish at infinity: $|u_m| \rightarrow 0$ as $m \rightarrow +\infty$. It is given by the formula:

$$v_m = (1 - r)u_m + ru_{m+1}, \quad m = 0, 1, 2, \ldots,$$  

which is obtained from (10.142a), (10.142b) by removing the right boundary. Finally, the operator $\overrightarrow{R}$ is defined on the linear space of functions $\{u_0, u_1, \ldots, u_{M-1}, u_M\}$ that satisfy: $|u_m| \rightarrow 0$ as $m \rightarrow -\infty$. It is given by the formula:

$$v_m = (1 - r)u_m + ru_{m+1}, \quad m = \ldots, -1, 0, 1, \ldots, M - 1,$$

$$\overrightarrow{R} = u_M,$$  

which is obtained from (10.142a), (10.142b) by removing the left boundary. Note that the spaces of functions for the operators $\overrightarrow{R}$ and $\overrightarrow{R}$ are defined on semi-infinite grids $m = 0, 1, 2, \ldots$ and $m = \ldots, -1, 0, 1, \ldots, M$, respectively.

None of the operators $\overrightarrow{R}, \overrightarrow{R},$ or $\overrightarrow{R}$ depend on $h$. We will show that the combination of all eigenvalues of these three auxiliary operators yields the spectrum of the family of operators $\{R_h\}$. In Section 10.5.1, we have, in fact, already computed the eigenvalues of the operators $\overrightarrow{R}$ and $\overrightarrow{R}$. For the operator $\overrightarrow{R}$, the eigenvalues are all