

We again conclude that the waves having different wavenumbers travel with different speeds, i.e., the propagation is accompanied by dispersion. Moreover, the long waves  $kh \ll 1$  happen to travel slower than with the speed  $-c$ . The phase speed of these waves may only approach  $-c$  as  $kh \rightarrow 0$ .

### 10.2.4 Predictor-Corrector Schemes

When constructing difference schemes for time-dependent partial differential equations, one can exploit the same key idea that provides the foundation of Runge-Kutta schemes for ordinary differential equations. This is the idea of introducing intermediate stages of computation, or equivalently, of employing the predictor-corrector strategy, see Sections 9.2.6 and 9.4. This strategy allows one to increase the order of accuracy beyond the one that would have been obtained if the original scheme was used by itself, i.e., with no intermediate stages. Besides, in the case of quasi-linear differential equations this strategy facilitates design of the so-called conservative finite-difference schemes that will be discussed in Chapter 11.

Let us recall the idea of the predictor-corrector approach using one of the simplest Runge-Kutta schemes as an example; this scheme will be applied to solving the Cauchy problem for a first order ordinary differential equation:

$$\frac{dy}{dt} = G(t, y), \quad y(0) = \psi, \quad 0 \leq t \leq T. \quad (10.60)$$

If the value of the solution  $y_p$  at the moment of time  $t_p = p\tau$  is already computed, then in order to compute  $y_{p+1}$  we first find the auxiliary quantity  $\tilde{y}_{p+1/2}$  using the standard forward Euler scheme in the capacity of a predictor:

$$\frac{\tilde{y}_{p+1/2} - y_p}{\tau/2} = G(t_p, y_p). \quad (10.61)$$

Subsequently, we apply the corrector scheme to compute  $y_{p+1}$ :

$$\frac{y_{p+1} - y_p}{\tau} = G(t_{p+1/2}, \tilde{y}_{p+1/2}). \quad (10.62)$$

The auxiliary quantity  $\tilde{y}_{p+1/2}$  obtained by scheme (10.61) with first order accuracy helps us approximately evaluate the slope of the integral curve at the midpoint of the interval  $[t_p, t_{p+1}]$  and thus obtain  $y_{p+1}$  by formula (10.62) with accuracy higher than that of the Euler scheme (10.61).

We have already mentioned in Section 9.4.2 that all these considerations will remain valid if  $y_p$ ,  $\tilde{y}_{p+1/2}$ , and  $y_{p+1}$  were to be interpreted as finite-dimensional vectors and  $G$ , accordingly, was to be thought of as a vector function. However, one can go even further and consider  $y_p$ ,  $\tilde{y}_{p+1/2}$ , and  $y_{p+1}$  as elements of some functional space, and  $G$  as an operator acting in this space.

For instance, the Cauchy problem:

$$\begin{aligned} \frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} &= 0, & -\infty < x < \infty, \quad 0 < t \leq T, \\ u(x, 0) &= \psi(x), & -\infty < x < \infty. \end{aligned} \quad (10.63)$$