Modeling of IC Debonding of FRP Strengthened Concrete Flexural Members

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Abstract: The presence of a fiber reinforced polymer (FRP) strengthening material bonded to the tension face of a reinforced concrete beam will restrict but not prevent the opening of intermediate flexural cracks due to applied loading. Test results indicate that displacements at the toe of flexural cracks create stress concentrations at the interface of the FRP laminate and the beam, leading to the development of localized interface cracks which typically propagate, under the effect of the load, to join the original flexural cracks and cause delamination of the FRP system. This type of FRP delamination is commonly termed intermediate crack (IC) debonding. In this paper the analytical models published in the literature are reviewed and found that these models do not correlate well with measured experimental results. This paper proposes an analytical model which characterizes the interface shear stress based on two distinct sources: the change in the applied moments along the length of the member, and stress concentrations at the intermediate cracks. The proposed model is compared to an experimental database and shown to predict extremely well most of the test results reported by other researchers. A parametric study, performed using the proposed model, indicates that the model varies with several important variables which are not captured by most of the existing models.

CE Database subject heading: Reinforced concrete; Prestressed concrete; Fiber reinforced polymers; Rehabilitation; Retrofitting; Analytical techniques.

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INTRODUCTION

The effectiveness of a FRP strengthening material bonded to the tension side of a flexural member is highly dependent on the bond stresses between the adherents and the member. Transfer of tension forces results in stresses that are a function of the applied loading conditions, as well as the geometry, configuration, and stiffness of the adherents and the adhesive. For reinforced/prestressed concrete, the distribution of bond stresses is more complex as a result of flexural and shear cracking disrupting the continuity of the FRP system. Bond stresses are induced due to the change of the internal moments along the length of the beam and by transfer of forces across cracks and at plate-end (Neubauer and Rostásy 1999). Bond failure, or debonding, usually occurs rapidly and can be initiated in several locations along the strengthened member. At the plate curtailment point, cracks due to stress concentrations may form as a result of the abrupt termination of the strengthening plate, or due to the influence of beam curvature on the material stiffness mismatch. This type of debonding is called plate-end (PE) debonding. For long-span flexural members, the stress concentrations due to the opening of flexural cracks are typically larger than the stress concentrations at the plate-end, and debonding advances from the intermediate cracks towards the support (IC debonding).

The first analytical models to describe the IC debonding process resulted from observations of the behavior of single lap-shear and double lap-shear specimens, which isolate a portion of a plated concrete beam with one flexural crack (Bizindavyi and Neale 1999, Maeda et al. 1997). Several researchers have derived expressions to predict the ultimate load of single lap-shear concrete specimens bonded with FRP based on the fracture energy of the concrete. The materials are usually assumed to be homogeneous and linear-elastic, the adhesive layer is assumed to be of negligible thickness and acts only as a medium to transfer the Mode II shearing loading, and the FRP plate and adhesive are assumed to be of uniform thickness. The solution depends on the relationship of interfacial shear stress to slip behavior of a concrete-FRP bonded joint, of which there have been several proposed relationships: linearly ascending (Täljsten 1994), linearly ascending and descending (Yuan et al. 2001, Brosens 2001, Chen and Teng 2001, Ulaga et al. 2003, Oehlers et al. 2004), using the power function (Brosens 2001), or bi-linear allowing for localized debonding (Leung and Tung 2001). Analytical methods exploring the presence of multiple flexural cracks along the length of the member have also been explored using a fracture based approach (Chen et al. 2005) and using relationships closely tied to the stiffness of the adhesive (Harmon et al. 2003).
Mechanics principles can be used to derive the shear stress in the FRP-concrete interface assuming perfect bond (Sebastian 2001). A discontinuous interfacial shear stress model has been proposed assuming the concrete remains elastic and using bi-linear relationships for longitudinal steel reinforcement (Wang and Ling 1998). A similar approach was also introduced which proposes design equations against bond failure utilized estimates of the internal lever arm and assumed equal tensile strains in the FRP and internal steel (Matthys 2000). Smear-crack finite element models have been used to develop design equations to predict IC debonding based on mechanics and fracture theories (Teng et al. 2004). National code documents provide design guidelines on IC debonding, based on fracture approaches (Ye et al. 2005, Fédération Internationale du Béton 2001, The Concrete Society 2004, Oehlers et al. 2006) and empirical relationships (ACI Committee 440 2002). This paper reviews the applicability of the various models reported in the literature and clearly identifies the need to develop a new analytical which accurately predicts the IC debonding strain of FRP strengthened reinforced and prestressed concrete beams. A review of the various models from the national codes can be found elsewhere (Rosenboom 2006).

ASSESSMENT OF CURRENT MODELS

In order to assess the IC debonding analytical models available in the literature, a database was constructed from 51 beams or slabs. In the database 47 beams failed due to IC debonding and 4 failed due to FRP rupture. The beams or slabs represent a wide cross-section of shapes and sizes, with depths varying from 150 mm up to 825 mm and shear span-to-depth ratios varying from 2.65 to 10.04. The steel reinforcement ratio varies from 0.34% to 2.7%. Three different FRP materials are found in the database: Carbon FRP, Aramid FRP, and Glass FRP which have been installed using both the wet lay-up method and through externally bonding a precured laminate using structural adhesive. The properties of the FRP vary significantly and represent both the laminate and fiber properties for wet lay-up systems. The beams or slabs were taken from 18 different literature sources. These sources, along with the full details of the database are given elsewhere (Rosenboom 2006). An assessment of the existing models from the literature was performed against the IC debonding database. Not every model was assessed, and only those models with clear failure criteria were examined. The models were followed exactly as described in the relevant literature except the following: 1) a FRP rupture check was performed whether the model explicitly called for one or not, and 2) the initial strain on the beam soffit due to the effect of dead load was included. A width factor \( k_b \) was included where advised by the model, and this is discussed at the end of this section. An assessment of the analytical models from the national code documents is provided in Rosenboom (2006).
**Models from Literature**

The empirical model by Maeda *et al.* (1997) was compared to the assembled IC debonding database. Regression analysis of test results obtained from double lap-shear specimens provided estimation of maximum strains in FRP at debonding failure. Two factors within the test setup helped in providing realistic results: 1) the presence of a layer of vinyl tape next to the "crack", and 2) the use of a double lap-shear configuration. Both of these factors have inhibited the formation of a concrete tooth in the specimen at failure which may significantly alter the behavior of the lap-shear specimen. The model tends to overestimate the debonding strain as the axial stiffness of the FRP decreases. The experimental versus predicted debonding strain is shown in Figure 1. Matthys (2000) identified a failure criterion to be checked called “transfer of forces”, where the derivative of the tensile envelope due to applied loading was determined and compared to the shear strength of the concrete. Two simple design equations were presented to determine the maximum shear force that could cause debonding before or after steel yielding, yet all the beams in the database failed after steel yielding. Although the predicted shear forces show good correlation to the experimental shear force, this is mainly due to the large disparity between the sizes of the beams in the database. The correlation could also be deceiving since a significant increase in debonding strain is not proportional to increases in applied shear force. An analytical model proposed by Leung *et al.* (2001) was assessed using the IC debonding database. One of the benefits of the model is its capability to predict the debonding strain at the main flexural crack at any various unbonded distances near the crack due to interfacial cracking. The value of acceptable localized debonding around the main flexural crack was assumed to be in the range of 25 percent of the height of the cross section of the beams. The experimental versus predicted debonding strains using the Leung *et al.* (2001) model are shown in Figure 2. The model is clearly underestimating the strain at debonding.

The model of Harmon *et al.* (2003) was assessed against the IC debonding database. The model takes into account the flexural crack spacing, and calculates the maximum force that can be developed in the FRP through an iterative procedure. One of the important variables in the estimation of the force in FRP at the critical crack location is the effective bond length, $L_{\text{eff}}$, which is equal to:

\[
L_{\text{eff}} = \sqrt{\frac{E_f t_f t_b}{G_a}}
\]  

(1)
where $E_f$ and $t_f$ are the elastic modulus and thickness of the FRP material, $t_b$ is the thickness of the bond (or adhesive) layer, and $G_a$ is the shear modulus of the adhesive. The effective bond length calculated using this equation is much less than the calculated value using other models (e.g. Chen and Teng 2001, or Oehlers & Seracino 2004). Figure 3 shows the correlation between the shear modulus of the adhesive and the effective bond length for various values of $E_f t_f t_b$ values (shown as $E_A$ in the plot). Also shown in the same figure is the value of $L_{\text{eff}}$ obtained using the Chen and Teng (2001) model using a concrete compressive strength of 33 MPa. In the Harmon et al. (2003) model the value of FRP force at the critical crack for IC debonding failure is heavily dependent on the value for effective bond length.

The fracture based model of Ulaga et al. (2003) was derived from an experimental program on double lap-shear specimens and assuming a linearly descending interface shear stress versus slip relationship. Similar to the other fracture based models, which are derived from test results on lap-shear specimens, the mean value of the model is conservative for the IC debonding database. Teng et al. (2004) developed a set of equations to predict the IC debonding resistance based on mechanics and fracture-based behavior. The calculation of one parameter in the model, the distance from the loaded end to the end of the cracked region ($L_{ce}$), was calculated using the following equation:

$$L_{ce} = a - \frac{M_{cr}}{M_{db}} a + s$$  \hspace{1cm} (2)

where $a$ is the shear span, $s$ is the distance from the center of the support to the FRP termination point, and $M_{cr}$ and $M_{db}$ are the cracking moment and nominal moment at the predicted debonding strain of the FRP strengthened section. Since $M_{db}$ can only be calculated once assuming a debonding strain, the model becomes iterative. Similar to conclusions proposed in the paper, the use of the more complicated mean model is not warranted since the design model has similar correlation. The experimental versus predicted debonding strains using the Teng et al. (2004) mean model are shown in Figure 4 to overestimate the debonding strain and provide poor correlation with the experimental database. The fracture based model was proposed by Chen et al. (2005) with a linearly descending shear stress versus slip relationship and assumed multiple flexural cracks to predict IC debonding. The model was derived from the behavior of a single lap-shear specimen having force applied to the FRP laminate from two directions, which simulates the behavior of the bonded joint between two flexural cracks. Two sets of equations are
presented, one which ignores the deformation in the concrete and the other equation which includes it. The equations which include the deformation in the concrete layer do not match well the behavior of the measured values of the flexural members included in the database, and have less correlation and are more conservative than the equations ignoring the concrete deformation. The ratio of the applied load on the FRP on either side of the concrete block (β) was assumed to be 0.8, a value which was used in the design example presented in the paper. The experimental versus predicted debonding strains ignoring the deformation in the concrete is shown in Figure 5. It should be noted that the equations developed by Chen et al. (2005) were derived from small scale specimens, however still compare well to the large scale specimens included in the database and are not overly conservative like earlier models (e.g. Chen and Teng 2001) as can be seen in the statistical analysis discussed later. This is most likely due to the more realistic boundary and loading conditions assumed in the derivation of the model, especially the nature of the applied load to both sides of the FRP laminate.

**Width Effect**

Many researchers have found that the width of the FRP plate has an effect on the shear stress distribution, and that increases in the width of the laminate are not proportional to increases in failure load (Brosens 2001, Chen and Teng 2001). Two possible justifications exist for the width effect: the size effect theory from fracture mechanics and the spreading out of forces in the concrete (Brosens 2001). Four different width factors were found in the literature and were assessed against the IC debonding database. A generalized equation for the width factor ($k_b$) can be written as:

$$ k_b = \alpha_b \sqrt{\frac{\chi_1 - \frac{b_y}{b_c}}{\chi_2 + \frac{b_y}{\varphi}}} $$

(3)

where $b_y$ is the width of the FRP laminate and $b_c$ is the width of the concrete surface. $\alpha_b$, $\chi_1$ and $\chi_2$ are empirical constants and were originally derived to be set equal to 1.0, 2.0 and 1.0 respectively (Brosens 2001). The value of $\varphi$ is set equal to $b_c$ in some models, and set equal to 400 mm in others. A table summarizing the four different width factors considered is shown in Table 1.

The beams in the IC debonding database were analyzed using the width factors described above and the IC debonding model of Leung and Tung (2001), which was assessed earlier. The application of the width factors $k_{b1}$...
and $k_{b2}$ do not provide for any better correlation and make the model more conservative compared to the experimental results. The application of the width factor $k_{b3}$ from The Concrete Society (2004) and Fédération Internationale du Béton (2001) also does not improve the correlation and has an adverse effect producing a less conservative prediction. The experimental versus predicted debonding strains for the original model, and after application of the four different width factors is shown in Figure 6. The model of Chen and Teng (2001) was analyzed in a similar manner and the same effect was observed. An additional size effect that should be acknowledged is the effect of the extra epoxy which is adjacent to the edge of the FRP laminate. A slight increase in strength is predicted due to the activation of concrete outside of the nominal dimensions of the FRP laminate. In experimental work, where small widths of FRP laminates are commonly used, this effect will be more pronounced than in field applications. Since the experimental work provides an upper bound width factor relationship, the width factor models of $k_{b1}$ or $k_{b2}$ would be preferable to $k_{b3}$. Both of these factors, however, deeply penalize the FRP configuration when approaching unity. It is not recommended to use width factors since there is little evidence of improvement of experimental to predicted correlation. However if they are used, the relationship shown as $k_{b4}$ in Table 1 is recommended.

**PROPOSED ANALYTICAL MODEL**

The analytical models presented in the literature do not provide good correlation to the collected database for IC debonding failures. Therefore this paper proposes a new analytical model based on mechanics and calibrated by the experimental database of IC debonding failures. Various failure criteria were examined in the construction of the model, and a check for possible rupture of FRP before debonding is also given. The initial strain in the beam soffit has been incorporated into the CFRP strain during calculation of the FRP system contribution to sectional strength. This strain is normally an additive quantity for prestressed concrete members and subtractive for reinforced concrete members. Considering a plated section with perfect bond along the plate-to-concrete interface the interface shear stress ($\tau_w$) can be evaluated as:

$$\tau_w(x) = \frac{d}{dx} \left[ K_p(x) \varepsilon_p(x) \right]$$  \hspace{1cm} (4)

where $d/dx[\varepsilon_p(x)]$ is the change in plate strain along the length of the beam $x$ and $K_p$ is the axial stiffness of the plating material per unit width. For an FRP plated concrete member with constant FRP configuration throughout the
length of the beam $K_p = nE_f$. Using traditional equilibrium and compatibility conditions along with the assumption that plane sections remain plane after deformation, an envelope of the axial strain in the FRP material can be determined for any given loading condition. The tensile strain in the internal steel reinforcement can also be calculated using the same approach and it can be determined at any given section after yielding of the steel.

Along with interface shear stress due to the applied loading, there exist normal (or peeling) stresses throughout the bonded length. At the plate termination point the peeling stresses result from a stiffness mismatch between the FRP and the concrete and cause an outward force. A short distance away from the plate-end, the peeling stresses due to applied loading become very small and occur in the opposite direction as clamping forces due to beam curvature. For this reason, the peeling stresses due to the applied loading away from the plate-end can be ignored in most cases, and controlled at the plate-end by using end anchorages. Since the peeling stresses will not significantly affect the behavior away from the supports, there exists only forward shear mode (Mode II) crack deformation on the FRP-to-concrete interface due to the applied loading. With the assumption of perfect bond, failure due to debonding will occur when the interface shear stress reaches the shear strength of the interface. Since the shear strength of the structural adhesive is usually greater than that of the concrete, failure will occur along the concrete interface.

FRP plated regular reinforced and prestressed concrete beams will exhibit flexural behavior when the shear span-to-depth ($a/d$) ratio is approximately 2.5 or greater, except in rare cases such as when there is exceptionally high prestress force. When flexural cracking occurs, the perfect bond assumptions are no longer valid at the location of the crack. At the toes of the flexural cracks stress concentrations form another set of cracking along the FRP-to-concrete interface, usually within the weak concrete layer. The stress concentrations at the intermediate cracks occur in two directions: 1) shear stress along the interface, and 2) peeling stresses due to aggregate interlock along the interface crack. As mentioned previously the normal stresses due to applied load are “clamping” stresses. These are in the opposite direction from the peeling stresses developed from stress concentrations, and are small in magnitude of both types of peeling stresses therefore they are disregarded in the current analysis. It is also possible to have wide crack openings due to flexure-shear cracks away from the maximum moment region, which could lead to high peeling stresses. It is assumed that the beam being strengthened in flexure has sufficient shear capacity to handle the higher loads and large shear crack deformations will not control the failure.
The interface shear stress at the toes of the flexural cracks due to stress concentration ($\tau_{sc}$) is related to the fracture energy of the weakest material at the interface. With quality workmanship and critical selection of an acceptable externally bonded FRP system, failure will propagate initially along the concrete interface. The profile of the shear stress due to stress concentration around the toe of a flexural crack is similar to a single lap-shear test, and can be approximated using Yuan et al. (2001) equations. As will be shown later, it is the magnitude of the interface shear stress which is of prime importance and this is not predicted using these equations.

The shear stress at the FRP-to-concrete interface ($\tau_i$) is induced from two sources: the applied loading ($\tau_w$) and stress concentrations at the toes of flexural cracks ($\tau_{sc}$), a philosophy which first appeared in Teng et al. (2004). For a distance $x$ from the support:

$$\tau_i(x) = \tau_w(x) + \tau_{sc}(x)$$

(5)

As $x$ increases along the length of a plated beam, the interface shear stress due to applied loading becomes larger proportional to the change in applied moment at the section. Alternatively, $\tau_{sc}$ is both additive and subtractive on either side of each flexural crack due to equilibrium of forces. The value will be maximum in high moment regions where the crack opening displacement has the greatest magnitude. A schematic of the applied moment, tensile force in FRP, and interface shear stress distribution is shown in Figure 7 for the loading configuration shown.

The interface shear stress due to applied loading ($\tau_w$) can be determined from a cracked section analysis along the length of the beam. A moment-curvature analysis of the plated section can be used to determine the tensile strain profile of the CFRP, from which the interface shear stress can be calculated from Equation 4. In Sebastian (2001) equations are developed which predict the interface shear stress due to the applied loading along the length of a beam. Sebastian (2001) showed that an assumption of perfect bond showed little error with experimental results, especially when coupled with smeared crack tension stiffening. A tensile strain envelope for one of the girders tested in Rosenboom (2006) is shown in Figure 8. The experimentally measured strain values are shown, along with three separate prediction methods: 1) the cracked section analysis program Response 2000 (Bentz 2000), 2) a flexural model described in Rosenboom (2006), and 3) the proposed predictive method which is discussed below. Note: for all of the analytical methods used, the strain envelope was created with the maximum value of tensile strain equal to the experimentally determined value. From the figure several conclusions can be drawn. The maximum change in tensile strain, and similarly the maximum interface shear stress, occurs at or near
the length along the beam at which yielding of the internal steel occurs (in this case at approximately 3200 mm from supports). The analysis also indicated that the predictions made from the flexural model, which is quite comprehensive, and the proposed method, which is relatively simple, correlate reasonably well with the measured values.

Therefore, it is proposed that the maximum interface shear stress due to applied loading ($\tau_{w\text{max}}$) be determined from the following equation, which is related to the increase of applied moment from the yielding moment ($M_y$) to the debonding moment ($M_{db}$). The equation predicts the maximum interface shear stress as in the region along the beam from the location of first yielding of the prestressing strands to location of debonding moment as:

$$\tau_{w\text{max}} = nE_f t_f \frac{E_{db} - E_{fy}}{a - x_y}$$

where $nE_f$ is the axial stiffness of FRP material per unit width, $E_{db}$ is the strain in the FRP at IC debonding failure at a moment of $M_{db}$, $E_{fy}$ is the tensile strain in the FRP at first yielding of internal tensile steel at a moment of $M_y$. $x_y$ is the distance from the support to the location of first yielding of internal tensile steel. For three and four point bending, $x_y$ is equal to:

$$x_y = a \frac{M_y}{M_{db}}$$

where $a$ is the shear span of the beam and is equal to the distance from the support to the section of maximum moment. For unsymmetrical loading, the shorter distance should be used. The distance $x_y$ can also be found easily for beams with other loading scenarios by considering the shape of the moment diagram. For example, for beams with uniform distributed loads, the distance $x_y$ can be determined as:

$$x_y = -\frac{L^2}{8M_{db}} \left( -\frac{4M_{db}}{L} + \sqrt{\left(\frac{4M_{db}}{L}\right)^2 - 16\left(\frac{M_{db}}{L^2}\right)M_y} \right)$$

Using equations similar to Equation 8 the interface shear stress due to applied loading can be calculated for the region between the supports and the location of flexural cracking ($x_{\text{cr}}$), and the region between the instance of first flexural cracking and yielding of the internal tensile steel as shown in Figure 9.
The total interface shear stress was compared to a criterion for failure. Since the majority of the bond stresses at the interface are from Mode II type loading, it is assumed that the failure criteria will be the shear strength of concrete. The concrete shear strength adopted in the proposed model is based on $\tau_{c,\text{max}} = 1.8 \times (f'_{t})$ where $f'_{t}$ is the tensile strength of concrete and can be assumed equal to $0.63(f'_{c})^{0.5}$ from Matthys (2000). Although this failure criterion gives higher values of the shear strength of concrete compared to many other models, it is believed that the shear strength of the FRP-to-concrete interface has a higher resistance to shear than plain concrete. The initial failure plane of beams which fail by intermediate crack debonding is at the FRP-concrete interface within the concrete surface. If sufficient surface preparation has been applied to open the pores of the concrete and expose the aggregates, the structural epoxy material will normally penetrate into the interface layer, making it able to resist more interface shear stress due to the higher shear modulus of the adhesive compared to the concrete. The analytical model was calibrated from a database of experimental intermediate crack (IC) debonding failures. The parameter that was calibrated was the maximum interface shear stress due to stress concentration ($\tau_{sc,\text{max}}$). This value was calibrated for two separate purposes: 1) for the creation of an analytical model that closely represents the experimentally observed behavior (mean model), and 2) an analytical model which is conservative and can be used in design (design model).

When the maximum interface shear strength due to applied loading ($\tau_{w,\text{max}}$) was calculated using the experimentally measured debonding strain for the beams in the database, it was determined that $\tau_{sc,\text{max}}$ calculated in this manner had no correlation to the strength of the concrete. Test results by the authors and others suggest that good correlation is found between the interface shear stress due to stress concentration and the moment ratio $M_y/M_{db}$. This is a result of the larger flexural crack opening displacement occurring as a result of having an applied moment much larger than the moment to cause yielding of the internal tensile steel.

The proposed equation for the maximum interface shear stress due to stress concentration ($\tau_{sc,\text{max}}$) which gives the best correlation between the analytical model and the experimental database was found to be:

$$\tau_{sc,\text{max}} = 2.15 \left(1.1 - \frac{M_y}{M_{db}}\right) \sqrt{f'_c}$$  \hspace{1cm} (9)

For design purposes, the following conservative equation may be used which provides a probability of exceedance of 5 percent:
\[ \tau_{sc,max} = 3 \left( 1.1 - \frac{M_y}{M_{db}} \right) \sqrt{f_c} \]  

(10)

The experimental versus predicted debonding strains for the proposed mean and design analytical models are shown in Figures 10 and 11 respectively. These figures provide evidence of the validity of the mean model to predict the test results and the conservatism of the design model.

In many of the beams collected in the experimental database, the observed failure mode was due to rupture of FRP and not IC debonding. The measured value of maximum strain near midspan prior to rupture was lower than that determined from FRP tensile tests, due to the presence of stress concentrations at the toes of the flexural cracks. Therefore it is believed that the strain due to these stress concentrations (\( \varepsilon_{sc} \)) caused rupture at a value of tensile strain assuming perfect bond (\( \varepsilon_{db} \)) lower than the ultimate rupture strain found during material testing (\( \varepsilon_u \)). As a result, the proposed model requires checking possible rupture of FRP due to stress concentrations at flexural cracks must be performed by comparing the summation of the strain and debonding (\( \varepsilon_{db} \)) and the strain due to stress concentration (\( \varepsilon_{sc} \)) to the ultimate rupture strain of the FRP material (\( \varepsilon_u \)) as follows: 

\[ \varepsilon_{db} + \varepsilon_{sc} \leq \varepsilon_u \]

(11)

For design, \( \varepsilon_u \) in the above equation should be reduced by an environmental reduction factor as proposed in ACI Committee 440 (2002).

The following is the recommended design procedure using the proposed model:

1. Calculate the moment resistance corresponding to first yielding of the tensile steel reinforcement, \( M_y \) and the corresponding strain level in the FRP material (\( \varepsilon_{fy} \)).

2. Assume a value for the strain in CFRP at failure due to intermediate crack (IC) debonding (\( \varepsilon_{db} \)).

3. Calculate the nominal moment resistance of the section (\( M_{db} \)) at debonding failure with the strain assumed in Step 2.
4. Determine the maximum interface shear stress due to the increase of the load from the yielding stage to the assumed failure stage, $\tau_{wmax}$ from Equation 6.

5. Determine the maximum interface shear stress due to stress concentration ($\tau_{scmax}$) using Equation 10.

6. Calculate the total interface shear stress ($\tau_i$) using Equation 5.

7. Revise the value of IC debonding strain until $\tau_i$ is equal to the failure criterion ($\tau_{cmax}$) which is related to the tensile strength of concrete as $\tau_{cmax} = 1.8 * [0.63(f'_c)^{0.5}]$.

8. Using the final value of strain in the CFRP after iteration ($\varepsilon_{db}$), calculate the maximum strain in the CFRP due to applied loading and stress concentrations ($\varepsilon_{scmax}$) using Equation 11.

9. If $\varepsilon_{scmax}$ is greater than $\varepsilon_u$ then failure will be due to rupture of the CFRP, and the nominal moment should be recalculated using the ultimate strain in the CFRP. $\varepsilon_u$ is the design rupture strain of the FRP material, after application of appropriate environmental reduction factors like the ones provided in ACI Committee 440 (2002).

**STATISTICAL ANALYSIS**

The proposed model as well as most of the analytical models discussed in this paper were statistically evaluated for comparison purposes. For each case, the mean ($\mu$) and standard deviation ($\sigma$) was determined for the variable $y = \frac{\text{(predicted IC debonding strain)}}{\text{(experimental IC debonding strain)}}$. The sample size in this statistical analysis is based on the 51 beams which failed due to IC debonding and included in the database as described in detail in Rosenboom (2006). The normal distribution was applied to the random variable $x$, using two tests (the Chi-squared and Kolmogorov-Smirnov) to determine the goodness of fit. The results of the statistical analysis are shown in Table 2. If the model did not pass the goodness of fit test, the corresponding cell has bold typeface. The majority of the models in the literature were conservative when calculated against an experimental database of beams and slabs of diverse sizes. The proposed mean model gives a mean value of 1.0, a correlation coefficient ($r^2$) of 76.3 percent and a standard deviation of 0.181. Figure 12 shows the variable $y$ which represents the ratio of the predicted debonding strain to the experimental debonding strain versus the normal distribution for the analytical models analyzed. There were several models which did not meet the 95 percent confidence level for either the Chi-
squared or Kolmogorov-Smirnov test. The model proposed by Harmon et al. (2003) did not match a normal distribution and was excluded in this figure.

PARAMETRIC STUDY

A parametric study was conducted using the proposed mean analytical model to predict intermediate crack (IC) debonding. Two types of concrete systems were examined: a reinforced concrete beam, and a prestressed concrete bridge girder. Various parameters were examined within the range of values found in the IC debonding database formulated in Rosenboom (2006), or within a range of values typically encountered in the field. The proposed mean model was examined in each case and was compared to the model by Teng et al. (2004). The prestressed concrete bridge girder examined in the parametric study was the girder EB1SB, tested by Rosenboom (2006). The reinforced concrete beam examined was beam B-08S by Kotynia and Kaminska (2003). As a reference point, the experimentally measured IC debonding strain corresponding to the appropriate parameter is shown as a point in each plot.

Applied Loading

The effect of the applied loading on the IC debonding resistance can be shown either by varying the shear span under three or four-point bending or by using different loading scenarios such as a uniformly distributed load. As discussed previously, there is a greater probability of the FRP system failing by plate end (PE) debonding once the shear span to depth ratio is below approximately 2.5. The study indicated that for beams loaded in three and four-point bending, the debonding resistance decreases as the shear span-to-depth ratio \((a/d)\) decreases as shown in Figure 13 for the prestressed concrete bridge girder. A similar plot can also be obtained for the reinforced concrete system. The figure also shows that the predicted resistance using the Teng et al. (2004) model provides a lower bound for low shear span to depth ratios, which gives conservative results as the \(s/d\) ratio is increased. Since the proposed analytical model for IC debonding is a function of the slope of the tensile strain in the CFRP strengthening material, for beams loaded using three or four-point bending the slope of this envelope is high near the point load. For structures loaded by uniformly distributed loads, the slope of the tensile envelope is less, so the debonding resistance in these cases would be greater. In the plots presented in the following sections the debonding resistance, assuming a uniform distributed load, is plotted along with the three or four-point bending resistance for comparison. In each case, the resistance assuming a uniform distributed load is higher.
Concrete Properties

Shear cracks during IC debonding propagate along the FRP-to-concrete interface along the concrete surface. Since the shear strength of the concrete is a function of the compressive strength, as the compressive strength of the concrete increases, the IC debonding resistance will increase. In both the proposed model and the model by Teng et al. (2004) this trend is applicable. The predicted debonding strain versus the concrete strength for the prestressed concrete system is shown in Figure 14. For very low concrete strengths, the failure mode in the proposed model begins to transition between IC debonding and crushing of concrete. The predicted debonding strain versus the concrete strength for the reinforced concrete system is shown in Figure 15. The model of Teng et al. (2004) describes the behavior of reinforced concrete better than prestressed concrete since it was calibrated with this type of system.

FRP Properties

The FRP axial stiffness per unit width \((nE_{ft})\) was examined as one of the parameters in the parametric study for the two types of structural systems. The strengthened systems were evaluated at FRP stiffnesses corresponding approximately to the range of values encountered in the IC debonding database: ranging from 10 to 500 kN/mm. This encompasses a wide range of FRP types including glass, aramid, and carbon of high, medium and low modulus of elasticity values. At low values of FRP stiffness, the likely mode of failure is FRP rupture for long span structures. With higher values of stiffness, the behavior becomes asymptotic to a minimum value of debonding strain, which is dependent on multiple factors. The predicted debonding strain versus the FRP axial stiffness per unit width is shown in Figure 16 for the reinforced concrete system. Similar to other parameters investigated, the model by Teng et al. (2004) gives better predictions for the reinforced concrete system; for the prestressed concrete system evaluated, the model predicts FRP rupture only with very low values of FRP stiffness. It should be noted that the rupture strain shown is for each of the FRP systems used for the girders examined and may vary if FRP material with different moduli of elasticity are examined.

Internal Steel Properties

The tensile steel reinforcement ratio has a significant effect on the debonding resistance due to its relationship with the yield strength and the ultimate strength of the section. This relationship is not captured by the model of Teng et al. (2004), where the strain at debonding is constant regardless of internal steel reinforcing ratio.
At low levels of internal steel reinforcing ratio, the interface shear stress due to applied loading is low; however, the shear stress due to stress concentration is high. This is mainly a result of the low value of the ratio $M_y / M_{db}$. As the internal steel reinforcement ratio is increased, two behaviors can occur. **Behavior 1**: the ratio of $M_y / M_{db}$ can approach unity and the predicted debonding strain will reduce, or **Behavior 2**: the FRP approaches rupture before the ratio of $M_y / M_{db}$ approaches unity. This is illustrated in Figure 17, where the predicted debonding strain is shown for the reinforced concrete system versus the internal steel reinforcing ratio. In Figure 17, for the beam loaded with a uniformly distributed load, **Behavior 2** from above occurs; and for the beam loaded in four-point bending, **Behavior 1** occurs. It should be noted that the maximum internal steel reinforcement ratio considered in the reinforced concrete system is 2 percent, approximately equal to the maximum limit prescribed in ACI Committee 318 (2005). It is possible to calculate the interface shear stress due to applied load if the predicted moment at debonding ($M_{db}$) is lower than the moment at yielding of the internal steel. This can be done through the calculation of a different interface shear stress block from the cracking moment as evidenced in Figure 9. This can be done using the cracking moment and the moment at yielding. This was not done in the parametric study, mainly because this behavior was not observed in the IC debonding database, where the range of predicted $M_y / M_{db}$ ratios for the mean model was between 0.64 and 0.9. The two different types of behavior discussed above are also observed in the parametric study of the prestressed concrete system. However, since the FRP axial stiffness is relatively low for this system, the FRP material ruptures before significant reduction of the predicted debonding strain has occurred due to **Behavior 2** discussed above. The ratio of $M_y / M_{db}$ predicted from the mean model for girder EB1SB was 0.66, one of the lower values observed in the IC debonding database. However, since the stiffness of the prestressed concrete girder after yielding of the prestressing strands is greater than that of a reinforced concrete beam, the interface shear stress due to the applied loading does not increase as rapidly as in the case of a reinforced concrete beam.

**Level of Prestress**

By increasing the effective force in the prestressing strands two main effects may occur: 1) the initial compressive strain in the soffit (prior to strengthening) becomes larger, and 2) the sectional moment at which the prestressing strands yield becomes smaller. The first behavior results in a beneficial effect, as the initial compressive strain can be added to the IC debonding strain. The second behavior has the opposite effect, as the interface shear stress due to applied loading becomes larger when the sectional moment at yield is reduced. For the
properties of the girder examined, the second behavior controls the relationship and the predicted debonding strain reduces as the effective prestressing force is increased. The equation by Teng et al. (2004) does not vary with changes in the effective prestress force, and gives conservative values of predicted IC debonding strain. The predicted debonding strain versus the ratio of effective prestress force to ultimate strength of the prestressing strands ($f_{pe}/f_{pu}$) is shown in Figure 18.

DESIGN EXAMPLE

A rectangular reinforced concrete beam with a concrete compressive strength of 34.5 MPa is 305 mm wide by 610 mm deep and is reinforced by three 24.8 mm diameter steel bars with a yielding strength of 414 MPa located at a distance 546 mm from the extreme compressive fiber. It is proposed to strengthen the 7310 mm long simply supported beam with two 305 mm wide Carbon FRP plies extended to a point near the supports. The ultimate tensile strength of the FRP material is 621 MPa, the modulus of elasticity is 37000 MPa and the thickness is 1.0 mm. The beam is loaded with a uniformly distributed load. The proposed design model described in this paper is used for the design with environmental reduction factors from ACI Committee 440 (2002) for CFRP with interior exposure.

The existing state of strain on the soffit was determined to be 0.00061 mm/mm. Through an iterative process and ACI Committee 440 (2002) stress block factors for concrete, the moment at first yielding of the internal steel was found to be 389.8 kN-m. By assuming a value of debonding strain of 0.0102 mm/mm, the debonding moment was determined to be 516.4 kN-m. The interface shear stress due to the applied loading was found using Equation 6 and Equation 8 to be 0.35 MPa. The interface shear stress due to stress concentration was found using Equation 10 to be 6.11 MPa. The summation of the interface shear stress due to stress concentration and due to applied loading was determined to be 6.46 MPa, which is less than the shear strength of concrete failure criterion $\tau_{c,max} = 1.8*\left[0.63(f'_c)^{0.5}\right]$ determined as 6.66 MPa. Upon revision of the assumed debonding strain, the failure criterion was matched when the debonding strain was set equal to 0.0109 mm/mm. The final value of strain in the CFRP after iteration was used to calculate the maximum strain in the CFRP due to the applied loading and stress concentrations using Equation 11. Since this value was less than the ultimate rupture strain of the CFRP material after application of the environmental reduction factors, the predicted failure is IC debonding. Using a strength reduction factor of 0.90, the nominal moment capacity of the strengthened section is 464.8 kN-m.
CONCLUSIONS

This paper proposes a new analytical model which accurately predicts the intermediate crack (IC) debonding strain for reinforced and prestressed concrete members flexurally strengthened with FRP materials. Several conclusions from the research can be made:

1. Since the majority of the current analytical models were derived using specific boundary conditions, loading configurations and test results from single and double lap-shear experiments, many of the models analyzed were conservative in nature and did not correlate well with the database.

2. From experimental observations and analytical modeling, it was found that the interface shear stress along the length of the strengthened member is directly related to two distinct sources: the applied loading and stress concentrations at the toes of flexural cracks.

3. From various analyses of the strengthened reinforced concrete member, it was found that the interface shear stress corresponding to the load increment from $M_y$ to $M_{lb}$ which can be easily calculated, compares well to more complex analyses and is useful in design.

4. Various width factors were analyzed in the construction of the analytical model and were found not to improve the correlation to the IC debonding database.

5. A parametric study was completed which showed that the proposed analytical model varies with shear span-to-depth ratio, longitudinal reinforcement ratio, prestress force and several other variables that are not captured in the current models from the literature. Future experimental and analytical research on IC debonding should consider different loading configurations.

ACKNOWLEDGMENTS

The authors would like to thank the generous contributions received from Paul Zia, Mina Dawood, Anthony Miller and Catrina Walter. The North Carolina Department of Transportation funded much of this research and should be acknowledged. The authors also thank the helpful and thoughtful comments provided by the anonymous reviewers.
REFERENCES


NOTATION

\( E_f \) = Modulus of Elasticity of FRP material
\( G_a \) = Shear modulus of adhesive
\( K_p \) = Axial stiffness of plating material per unit width
\( L \) = Length of beam
\( L_{ee} \) = Distance from loaded end to end of cracked region
\( L_{eff} \) = Effective bonded length
\( M_{cr} \) = Flexural cracking moment of strengthened section
\( M_{db} \) = Nominal moment capacity at intermediate crack (IC) debonding
\( M_y \) = Moment at first yielding of internal tensile steel
\( a \) = Shear span
\( b_c \) = Width of concrete surface
\( b_f \) = Width of FRP laminate
\( d \) = Distance from extreme compressive fiber to centroid of tensile reinforcing steel
\( f'_{b} \) = Concrete tensile strength
\( f'_{c} \) = Concrete compressive strength
\( f_{pe} \) = Effective prestress force
\( f_{pu} \) = Ultimate tensile strength of prestressing strand
\( h \) = Height of section
\( k_{bi} \) = Width factor
\( l_{db} \) = Unbonded distance near flexural crack due to interfacial cracking
\( n \) = Number of layers of FRP
\( r^2 \) = Correlation coefficient
\( s \) = Distance from center of supports to FRP termination point
\( t_f \) = Thickness of one layer of FRP material
\( t_b \) = Thickness of adhesive (bond) layer
\( x \) = Distance along beam from left support
\( x_{cr} \) = Distance from support to location of first flexural cracking of concrete
\( x_y \) = Distance from support to location of first yielding of internal tensile steel
\( y \) = Random variable in statistical analysis
\( a_b \) = Calibration factor used in width factor
\( \beta \) = Ratio of applied load to the FRP on either side of concrete block
\( \varepsilon_{db} \) = Tensile strain in FRP at failure due to IC debonding
\( \varepsilon_{fy} \) = Tensile strain in FRP at first yielding of internal tensile steel reinforcing
\[ \varepsilon_{sc} = \text{Tensile strain in FRP due to stress concentration} \]

\[ \varepsilon_u = \text{Ultimate rupture strain of FRP material after application of environmental reduction factor} \]

\[ \phi = \text{Width of concrete empirical constant used in width factor} \]

\[ \tau_{c_{\text{max}}} = \text{Shear strength of concrete failure criterion} \]

\[ \tau_i = \text{Interface shear stress} \]

\[ \tau_{sc} = \text{Interface shear stress due to stress concentration} \]

\[ \tau_{i_{\text{max}}} = \text{Maximum interface shear stress due to stress concentration} \]

\[ \tau_w = \text{Interface shear stress due to applied load} \]

\[ \chi_1 = \text{Calibration factor used in width factor} \]

\[ \chi_2 = \text{Calibration factor used in width factor} \]
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<tr>
<th>Width Factor</th>
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<th>$\alpha_b$</th>
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* Width factors are not recommended. If used, the proposed equation should be used.
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Tensile force in FRP

\[ T_{\text{FRP}} = T_{\text{FRPv}} + T_{\text{FRPc}} \]

Interface shear stress

\[ \tau_i = \tau_w + \tau_{sc} \]

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