Experimental and Analytical Modeling of Concrete-Filled FRP Tubes Subjected to Combined Bending and Axial Loads

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Abstract

This paper presents test results of an experimental program and proposes an analytical model to describe the behavior of concrete-filled fiber reinforced polymer (FRP) tubes subjected to combined axial compression loads and bending moments. The experimental program included ten specimens subjected to eccentric axial loads, two specimens tested under concentric axial loads and two specimens tested in bending. Glass-FRP tubes with two different laminate structures were considered. Axial load - bending moment interaction curves are presented. The paper presents an analytical model, which accounts for variable confinement of concrete and gradual change of the bi-axial state of stresses developed in the tube as the eccentricity changes. The model utilizes the classical lamination theory for the FRP tubes and accounts for their gradual reduction of stiffness as a result of the progressive failure of different FRP layers. A parametric study was conducted to evaluate the effects of diameter-to-thickness ratio and laminate structure of the tube including fiber proportions in the axial and hoop directions. The study evaluated the confinement as affected by the eccentricity of the applied axial load as well as the influence of the FRP laminate structure. Research findings indicate significant increase of the flexural strength by increasing the ratio of fibers in axial direction. Increasing the ratio of fibers in the hoop direction increases the axial compressive strength of concrete-filled thin tubes.

Key words: FRP, tubes, concrete, beam, column, combined loading, confinement, interaction curve.

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Introduction

Concrete-filled fiber reinforced polymer (FRP) tubes provide a new and attractive use of composite materials in several applications including piles, columns, bridge piers, poles and highway overhead sign structures. Traditional pile materials including steel, concrete, and timber have limited service life and high maintenance costs, specially if they are used in harsh marine environments (Lampo et al., 1998). It has been estimated that repair and replacement of piling systems costs the U.S. over $1 billion annually (Lampo, 1996). High repair and replacement costs have led North American highway agencies and researchers to investigate the feasibility of using composite materials for civil engineering infrastructures including bridge pile foundations (Iskander and Hassan, 1998). FRP tubes provide a permanent, non-corrosive, lightweight formwork for the concrete and reinforcement element at the same time. The laminate structure of the composite tube can be controlled to provide different proportions of strength and stiffness in the longitudinal and transverse directions, depending on the application and nature of loading. Under axial loads, the FRP tube confines the concrete by reducing its lateral expansion, therefore, increases its ultimate strain and strength. Several researchers have studied the structural behavior of concrete-filled FRP tubes under axial loads (Fam and Rizkalla 2001(a) and (b) and Mirmiran and Shahawy 1997). Flexural behavior of these members was also studied by Fam and Rizkalla (2002) for Glass-FRP tubes and by Karbhari et al (1998) for carbon-FRP tubes. Seible (1996) proposed the concrete-filled CFRP tubes for different bridge systems. Research related to the behavior of concrete-filled FRP tubes under combined axial load and bending moments is still very limited. Mirmiran et al (2000) studied both thin and thick wall tubes to investigate under- and over-reinforced sections subjected to a constant axial load and increasing bending, using transverse loads, in order to compare the behavior of concrete-filled FRP tubes with that of conventional prestressed piles. The study concluded that bond failure is not a concern in members subjected to combined bending and axial loads and also over-reinforced sections were recommended for design due to their higher strength and stiffness. Teng et al (2002) have conducted a theoretical study on reinforced concrete circular members wrapped with FRP sheets and subjected to combined bending and axial loads in order to establish the interaction diagrams using different confinement models proposed by different researchers. The study emphasized the significant effect of confinement as the axial load increases and also showed almost no effect of confinement on the pure bending strength, which was also reported earlier by Fam (2000) based on beam tests using concrete-filled FRP tubes.

Research Significance

This paper provides experimental results of large-scale concrete-filled FRP tubes using two different laminate structures tested under bending, concentric and eccentric axial loads, in order to establish the interaction diagrams. The FRP tube is the sole reinforcement in both longitudinal and circumferential directions, therefore, the analytical modeling accounts for the gradual change of the state of bi-axial stresses developed in the tube as the eccentricity of the axial load changes. The lamination theory and the method of ultimate laminate failure are adopted to account for the gradual reduction of stiffness due to the progressive failure of different FRP layers. Failure of the FRP tubes under combined axial compressive and hoop tensile stresses is detected by the Tsai-Wu failure criteria. Three different confinement mechanisms of concrete are examined, including an upper bound full confinement model which is based on a fixed level of confinement, independent of the eccentricity of the applied load, a lower bound unconfined concrete model, and a partial confinement model, which is variable and function of the eccentricity of the load. FRP tubes with different wall thickness and various proportions of strength and stiffness in the axial and hoop directions have been evaluated using parametric study to examine the effects of both reinforcement ratio and laminate structure on the interaction diagrams of concrete-filled FRP tubes subjected to combined bending and axial loads.

Experimental Program

The experimental program included testing of concrete-filled Glass-FRP (GFRP) circular tubes under concentric and eccentric axial loads as well as under pure bending. Two different laminate structures were used for the GFRP tubes. Table 1 provides details of test specimens including the type of loading (bending, axial compression and combined bending and axial compression), the type of GFRP tube, the eccentricity of the axial load, *e* (for the eccentrically loaded columns) the span of the beam (L) as well as the height of the concentrically and eccentrically loaded columns (H), the average compressive strength f_c' of the concrete used to fill the tubes based on standard cylinder tests, the measured axial load P_n and bending moment M_n at failure. The GFRP tubes had 51 percent fiber volume fraction and were fabricated using the filament winding method. Table 2 provides details of the two types of GFRP tubes including the diameter, wall thickness, stacking sequence of different layers including the angle of the fibers and fiber/matrix types. Table 2 also provides the effective mechanical properties of the laminates based on the classical lamination theory including the effect of progressive laminate failure. Type I tubes have almost equal fiber percentages, oriented at 3 and 88 degrees with the longitudinal direction, while Type II tubes have 70 percent of the fibers oriented at ± 34 degrees and 30 percent at 80 degrees with the axial direction. Coupons from the longitudinal direction of the tubes were tested in tension in order to verify the prediction by the lamination theory and progressive failure method. The experimental and predicted stress-strain behavior in the longitudinal direction is given in Fig. 1 for the two types of tubes.

- **Fabrication of specimens**

The hollow GFRP tubes were placed in an inclined position on a steel frame. Wooden plugs were installed at the top and bottom ends of the tubes. High slump concrete was pumped from a ready mix truck into the tube through a hole in the upper plug. External vibrator was fixed to the steel frame. The concrete mix design incorporated an expansive agent to overcome any possible shrinkage during the curing process. After sufficient curing time, test specimens were cut from the long tubes using a diamond blade saw.

- **Beam specimens**

Table 1 provides details of B1-I and B1-II beam specimens using FRP tubes Types I and II. The specimens were tested using four-point bending as shown in Fig. 2(a). Two identical specimens were tested for B1-I and B1-II. The span of the beams was 5.5 m while the distance between the two applied loads was 1.5 m. Specimens were instrumented within the constant moment region to measure the longitudinal and circumferential strains along the depth. Mid-span deflection and applied load were also measured. The beams were tested to failure to determine the flexural capacity M_n. Fig. 3 shows the load-deflection behavior of the two beams. The behavior indicates that the cracking load is quite low in comparison to the ultimate load and behavior is almost linear, after cracking, up to failure.

- **Concentrically loaded column specimens**

Table 1 provides details of C1-I and C1-II column specimens using FRP tubes, Types I and II. Both specimens were tested under concentric axial load to failure, as shown in Fig. 2(b), to provide the axial strength P_n. Two identical specimens were tested for C1-I. Axial and circumferential strains were measured using both displacement and strain gauges along the perimeter of the tube at mid-height. Fig. 4 shows the axial load-axial strain behavior of the columns based on an average value of three strain gauges and an average value of three displacement gauges. The two types of gauges were staggered around the perimeter, 60 degrees apart. The unconfined concrete had a relatively high compressive strength $f_c^{'}$, and therefore, the concrete lacks the post-peak strain softening response typically observed in unconfined low strength concrete stress-strain curve. Due to the brittle nature of this concrete, the load reached a peak value, corresponds approximately to $f_c^{'}$ and dropped slightly, in C1-I specimens, after development of few major internal cracks. Beyond this stage, the behavior of C1-I specimens showed plastic behavior and the peak load was gradually recovered until the tube was fractured and the columns failed. For C1-II specimen, larger drop in the load was observed without noticeable recovery, mainly due to the laminate structure of Type II tubes, which has low stiffness in the hoop direction and relatively high Poisson's ratio value. In a typical situation, where low or normal strength concrete is confined, internal cracks are very uniform and well distributed within the concrete mass, resulting in bi-linear load-strain behavior for FRP tubes with adequate stiffness (Fam and Rizkalla 2001(a) and Mirmiran and Shahawy 1997). In this test, few internal cracks occurred due to the brittle nature of the relatively high strength concrete used. The measured axial load capacity of C1-II was slightly higher than C1-I in spite of the better confinement effectiveness of Type II tubes in comparison to Type I tubes. This is attributed to $f_c^{'}$ value of the concrete filling, which was higher in C1-II (67 MPa) than in C1-I (60 MPa). In general, both columns exhibited small confinement effect, mainly due to the brittle nature of the concrete core, which resulted in limited expansion in the transverse direction under axial loading, and consequently, low confinement pressure. Also, axial loading of the FRP tube and the design of the FRP laminate, which provided a limited fraction of the fibers in the hoop direction, have contributed to the low confinement pressure and the limited gain in axial strength in this case.

- **Eccentrically loaded column specimens**

Table 1 provides details of the eccentrically loaded column specimens (BC1-I to BC5-I) of Type I tubes as well as (BC1-II to BC5-II) of Type II tubes. Rigid steel caps were installed at the top and bottom of the specimens, as shown in Fig. 2(c), to allow for variation of the eccentricity of the applied axial load, therefore, provide different combinations of axial loads P_n and bending moments M_n at failure. Longitudinal and circumferential strains as well as the net lateral deflection at mid-height were measured. Due to the nature of the load application, the applied axial force and bending moment were coupled. The total maximum moment M_n at mid-height, which is reported in Table 1 for all eccentrically loaded column specimens, is composed of the primary moment, based on the initial eccentricity, and the secondary moment due to the lateral deflection at failure at mid-height. Table 1 also presents the eccentricity based on the final M_n and P_n, which ranged from 55 to 839 mm for Type I tubes and from 11 to 329 mm for Type II tubes. These values are equivalent to eccentricity e - to - outer diameter D_o ratios of 0.169 to 2.574 for Type I and 0.034 to 1.028 for Type II tubes. Fig. 5 shows the axial load-bending moment interaction diagrams for Type I and II specimens. The behavior reflects very well the transition from tension to compression failure through the balanced point. The interaction diagrams are discussed in more details in the analytical modeling section.

- **Failure modes**

Fig. 2 shows the different failure modes of some of the test specimens. Beam specimens B1-I and B1-II failed by rupture of the fibers in the tension side within the constant moment region in a similar fashion to the eccentrically loaded column specimens, which failed in tension as shown in Fig. 2 (c). The axial strains measured on the compression side of the tubes of beams B1-I and B1-II were 0.0072 and 0.0058 respectively (with no sign of compression damage), when the beams failed in tension at 0.02 and 0.15 axial tensile strains, respectively. The C1-I and C1-II column specimens failed by fracture of the tube under a state of bi-axial stresses including axial compressive and hoop tensile stresses as shown in Fig. 2 (b). C1-II column also showed minor local buckling of the tube under axial compression. The axial stresses in both types of specimens is a result of the direct axial loading on the tube, while the hoop stresses results mainly from expansion of the concrete inside the tube under high axial stresses. The behavior of C1-II specimen suggests that concrete expansion did not engage the FRP tube to produce significant level of confinement. Eccentrically loaded column specimens failed either in tension or compression, depending on the eccentricity of the applied load. BC1-I, BC2-I, BC3-I, BC1-II, and BC2-II failed in tension by rupture of the fibers, similar to the beams B1-I and B1-II, while BC5-I, BC3-II, BC4-II and BC5-II failed in compression by crushing of the fibers in the compression side as shown in Fig. 2(c). BC4-I had a balanced failure including rupture of fibers in tension side, almost simultaneously with crushing of the fibers in the compression side. Both tension and compression failure modes are illustrated in Fig. 2(c) for specimens of both Type I and Type II tubes.

Analytical Modeling

The objective of the proposed analytical model is to establish the axial load-bending moment interaction diagram of concrete-filled FRP tubes. The model is based on the equilibrium and strain compatibility approach. The layer-by-layer method is used for the integration process of the stresses over the cross-section of the member, in order to determine the ultimate axial load and bending moment acting on the cross-section under different eccentricities. The classical lamination theory is used to establish the effective stress-strain curves of the laminate of FRP tubes in the axial and hoop directions, utilizing the progressive failure approach of different layers of the laminate. Failure of the laminate is determined by the Tsai-Wu failure criteria (Daniel and Ishai 1994). For the stress-strain curve of concrete in the compression zone, three different confinement mechanisms are examined, including an upper bound representing the full confinement model, which represent fixed level of confinement, independent of the eccentricity of the applied load, a lower bound unconfined concrete model, and a partial confinement model, which is variable and dependent on the eccentricity of the load.

- **Confinement of concrete**

Three different mechanisms simulating the behavior of concrete in the compression zone can be categorized as follows:

***Full confinement mechanism* (*Upper bound*)**

In this case, the stress-strain curve of the confined concrete can be determined using any of the confinement models available in the literature for the case of pure axial loading condition. The analysis in this case utilizes the same confined stress-strain curve, shown in Fig. 6(a), in the compression zone, for the full range of eccentricity, to establish the interaction diagram. The confinement model by Fam and Rizkalla (2001 b) has been adopted in this analysis. The model is well suited for this case because it accounts for the bi-axial state of stress developed in the tube, including the axial compressive stresses, which results from the composite action, and the hoop tensile stresses resulting from confinement. The model adopts Tsai-Wu failure criteria to target the failure point of the tube. The ultimate confined strength and corresponding strain are

f'_{cc} and ε'_{cc} respectively. For FRP tubes with adequate stiffness, the model is almost bi-linear with the transition point near the unconfined peak strength, f'_c.

Unconfined concrete model (Lower bound)

In this case, an unconfined stress-strain concrete model, shown in Fig. 6(a), is adopted in the compression zone for the full range of eccentricity to establish the interaction diagram. In this analysis the model by Popovics (1973) is used due to its accurate simulation, especially for the strain softening behavior. In this model, the stress f_c at a given axial strain ε_c is given as a function of the unconfined strength f'_c and the corresponding strain ε'_c as follows:

$$f_c = \frac{f'_c \, x \, r}{r - 1 + x^r} \tag{1}$$

Where $x = \varepsilon_c / \varepsilon'_c$ and $r = E_{co} / \left(E_{co} - E_{sec}\right)$. E_{co} is the tangent elastic modulus of unconfined concrete, and can be estimated as $5000\sqrt{f'_c}$ (MPa). E_{sec} is the secant modulus of unconfined concrete and can be estimated as f'_c / ε'_c.

The curve could be terminated at 0.003 strain, which is the ultimate strain, specified by ACI 318-02, or could be extended to strain ε_{cco}. Fam and Rizkalla (2002) have shown that for the case of pure bending the effect of confinement on concrete strength is insignificant, however, the ductility and strain of concrete are increased significantly beyond 0.003. It is also well established that failure of the system is normally governed by failure of the FRP tube before complete failure of the concrete inside. Therefore, ε_{cco} is assumed equal to the ultimate compressive strain of the FRP tube in the axial direction.

Variable confinement mechanism

The proposed variable confinement model assumes that the confinement level of concrete is gradually reduced as the eccentricity of the axial load increases. This mechanism is very representative to observed behavior. Test results indicated that increasing the eccentricity results in a strain gradient that subject large part of the cross-section to tensile strains, which would significantly reduce the level of confinement. Fig. 6(b) shows the variable stress-strain curve of concrete, which ranges from the upper limit of the fully confined concrete (zero eccentricity) to the lower limit of the unconfined stress-strain curve with extended ductility for the case of infinite eccentricity (pure bending). The proposed model assumes that the initial ascending part of the curve is similar for both unconfined and confined concrete as observed by several researchers [Fam and Rizkalla (2001 a) and Samaan et al (1998)]. The part of the curve beyond the peak point of unconfined strength f_c', is variable and dependent on the eccentricity of the applied load. For a given general eccentricity e, the ultimate strength of concrete $\overline{f_{cc}}$, is calculated as a function of the fully confined strength f_{cc}' and the unconfined stress f_{cco} at ultimate strain, from the following proposed equation:

$$\overline{f_{cc}} = \left(f_{cc}' - f_{cco}\right)\left[\frac{D_o}{D_o + e}\right] + f_{cco} \qquad (2)$$

Where D_o is the outer diameter. This expression satisfies the upper and lower limits. For a case of pure axial load (e = 0), $\overline{f_{cc}} = f_{cc}'$ and for the case of pure bending ($e = \infty$), $\overline{f_{cc}} = f_{cco}$.

Fam and Rizkalla (2001 b) have shown that for the case of axial load (e = 0), a bi-axial state of stress is developed in the FRP tube and therefore, a bi-axial strength failure criteria such as Tsai-Wu should be used to detect failure. Fig. 6(c) shows the stress path (point 0 to 1) during the loading history under pure axial load. By increasing the eccentricity, less confinement is generated, and therefore, less hoop tensile stresses are developed and the stress path would gradually shifts from (point 0 to 1) to (point 0 to 2). As the eccentricity reaches infinity (pure bending), the stress path would be from point 0 to 3. Therefore, the path between points 1 and 3 on Fig. 6(b) corresponds to the path between points 1 and 3 on the failure envelope in Fig. 6(c). Accordingly, the locus of failure points between f'_{cc} and f_{cco} in Fig. 6(b) is analogous to Tsai-Wu failure envelope and is approximated as elliptical. The strain $\overline{\varepsilon_{cc}}$, which corresponds to the strength $\overline{f_{cc}}$ and ranges from ε'_{cc} to ε_{cco}, is calculated from the following elliptical equation:

$$\overline{\varepsilon_{cc}} = \left(\varepsilon_{cco} - \varepsilon'_{cc}\right)\sqrt{1-\left(\frac{\overline{f_{cc}} - f_{cco}}{f'_{cc} - f_{cco}}\right)^2} + \varepsilon'_{cc} \tag{3}$$

The shape of the curve between f'_c and $\overline{f_{cc}}$ ranges from approximately linear at $e = 0$ to the non-linear function of Popovics at $e = \infty$, which is given in Equation 1, depending on the value of $\overline{f_{cc}}$. In order to allow for the gradual and smooth transition between the upper and lower bounds, a modified expression of Equation 1 is used, as shown in Equation 4, where a shape parameter α has been introduced.

$$\frac{\overline{f_{cc}}}{f'_c} = \frac{\bar{x}(\alpha r)}{(\alpha r) - 1 + \bar{x}^{(\alpha r)}} \tag{4}$$

Where $\bar{x} = \overline{\varepsilon_{cc}} / \varepsilon'_c$. Knowing $\overline{f_{cc}}$ and $\overline{\varepsilon_{cc}}$, Equation 4 can be solved for α. The full curve between f'_c and $\overline{f_{cc}}$ can then be established using the same equation, Equation 4, to get different points (stresses and corresponding strains) using the obtained value of α. A trial and error procedure would be used to solve Equation 4 for α, due to its complex nature. Fig. 7 shows a family of curves representing the solution of the equation for (α r), for a wide range of $\bar{x} = \overline{\varepsilon_{cc}} / \varepsilon_c^{'}$.

- **Section analysis for establishing the interaction diagram**

Concrete-filled FRP circular tubes under axial load and bending moment are subjected to variable axial stresses along the depth of the member, which are also distributed over an area of variable width. The reinforcement consists of the FRP tube, which is a continuous surface. Due to the complex nature of the geometry and stress distribution, numerical integration of stresses is used to calculate the forces, utilizing the layer-by-layer approach [Fam 2000]. The section is subdivided into several horizontal layers as shown in Fig. 8(a). The reinforcement within each layer consists of the portion of the FRP tube available within the depth of the layer. Fig. 8(a) shows the original and idealized sections used for analysis. Linear strain distribution and full composite action are assumed. The axial compressive and tensile stresses in the FRP tube are based on the effective stress-strain curves of the FRP laminate in the axial direction. The compressive stresses in concrete are based on the different confinement mechanisms. The analysis is performed for a given eccentricity e, by assuming values for the extreme compressive strain, ε_c, and a neutral axis depth c. The strains $\varepsilon(i)$ and corresponding stresses in FRP, $f_f(i)$, and concrete, $f_c(i)$, are calculated and used to determine the compressive forces in the FRP and concrete, $CF(i)$ and $CC(i)$ respectively, as well as the tension forces in FRP, $TF(i)$, at each layer, i. The resultant of all the internal forces, N, as well as the corresponding moment, M, are calculated and used to calculate the eccentricity [$e = M / N$]. If the calculated eccentricity is different from the value assumed initially, the neutral axis depth c is changed and the process is repeated until the calculated eccentricity is equal to the assumed one, and (M, N) are obtained as shown in Fig. 8(b) as point "1". The whole process is repeated for different values of the extreme compressive strain ε_c, until the maximum values of M and N are obtained. $(M, N)_{\max}$ are shown in Fig. 8(b) as point "2". These values normally correspond to reaching the ultimate compressive or tensile strength of the material in most cases. Different combinations of $(M, N)_{\max}$ are used to establish the full interaction diagram for the section at different eccentricities.

- **Stress-strain curves of the FRP tube**

The classical lamination theory [Daniel and Ishai, 1994] is used to calculate the effective elastic modulus of the multi-layer laminate of the tube in the axial and hoop directions assuming a flat membrane element subjected to in-plane forces. This assumption is valid since all The ultimate laminate failure approach (ULF), which is based on progressive failure of different layers, is used to estimate the complete stress-strain curve of the laminate, including the gradual reduction of stiffness as shown in Fig. 8(c). The predicted stress-strain curves of the FRP tubes in the axial direction are shown in Fig. 1 and used in the section analysis, which is illustrated in Fig. 8(a).

- **Proposed analysis procedure**

The analysis procedure, using the variable confinement model, can be summarized as follows: (a) Use any available confinement model, such as the one by Fam and Rizkalla (2001 b), to establish the stress-strain curve of confined concrete under axial compression, including the values of f'_{cc} and ε'_{cc} at ultimate as shown in Fig. 6(a).

(b) Use the model by Popovics (1973), given in Equation 1, to establish the unconfined stress-strain curve of unconfined concrete as shown in Fig. 6(a). The curve is terminated at axial strain ε_{cco} equals to the ultimate axial compressive strain of the FRP tube, obtained from lamination theory. The axial stress corresponding to ε_{cco} is f_{cco}.

(c) At each eccentricity *e*, Equations 2 and 3 are used to calculate the ultimate strength $\overline{f_{cc}}$ and corresponding strain $\overline{\varepsilon_{cc}}$.

(d) Using $\overline{f_{cc}}$ and $\overline{\varepsilon_{cc}}$, Equation 4 or Fig. 7 can be used to calculate the shape factor α.

(e) Using α, Equation 4 can be used again to establish the full stress-strain curve (f_{cc} - ε_{cc}) between $f_c^{'}$ and $\overline{f_{cc}}$ as shown in Fig. 6(b). The part of the curve before $f_c^{'}$, is assumed similar to the unconfined curve.

(f) The obtained stress-strain curve of concrete is used in the section analysis for that particular eccentricity, to obtain a point on the interaction diagram as shown in Fig. 8.

Verification of the Model

The model has been applied to the test specimens, from Types I and II, in order to predict the interaction diagrams, using the procedure described above. Also, the load-strain behavior of the columns under pure axial load as well as the load-deflection behavior of the beams under pure bending, have been predicted. Fig. 1 shows the stress-strain curve of the FRP tubes of Types I and II under axial tension, based on coupon tests as well as the prediction using the lamination theory. Fig. 1 also shows the measured FRP longitudinal ultimate tensile strains in the beam tests of the concrete-filled FRP tubes, which indicate that coupon tests could underestimate the tensile strength of circular filament-wound FRP tubes. In general, the prediction using the lamination theory shows good agreement with test results, however, it underestimated the longitudinal ultimate strains of the tubes by 1.4 and 18.7 percent for Type I and Type II tubes respectively. Similarly, the strength and stiffness of the tubes under axial compression and hoop tension have been predicted and given in Table 2.

Fig. 9(a) shows the predicted stress-strain curves of concrete for Type I specimens, including the full confinement model (upper bound), the unconfined model (lower bound) and the variable confinement model, which results in a different curve for different eccentricities. Fig. 9(b) shows the predicted stress-strain curves of concrete for Type II specimens, including the upper and lower bounds. It is evident that the tubes in Type II specimens provide low level of confinement, which is also confirmed by the measured load-axial strain behavior of column C1-II in Fig. 4. Low confinement is attributed to the laminate structure of Type II tubes, which resulted in low stiffness in the hoop direction as well as a high value of Poisson's ratio, and accordingly, resulted in separation between the concrete core and the tube and delay of the confinement mechanism.

- **Interaction diagrams**

Using the predicted stress-strain curves of the FRP tube and concrete, the section analysis using layer-by-layer approach has been conducted in order to establish the full interaction diagram for Types I and II specimens. Fig. 5 (a) and (b) shows the experimental results as well as the predicted interaction diagrams using different confinement mechanisms, for specimens of Types I and II respectively. Fig. 5(a) shows that the variable confinement mechanism provides the best prediction. It however over-estimated the moment at the balanced point by 1.6 percent and under-estimated the axial load by 17.8 percent. It is also noted that the full confinement mechanism provides reasonable prediction of the interaction diagram, however, it overestimates the bending capacity under low axial loads or under pure bending. The unconfined concrete model significantly underestimated the interaction diagram. However, under pure bending, the unconfined concrete model with extended strain softening predicts the bending capacity very well, which was also reported by Fam and Rizkalla (2002). Lack of confinement in bending is attributed to the small area of concrete in compression as well as the strain gradient. The unconfined concrete model with ultimate strain limited to 0.003 underestimates the bending capacity. Fig. 5 also shows the contribution of the plain concrete core (without the effect of the tube) to the interaction diagrams. Fig. 5(b) shows that the full confinement model provides good agreement with the test results, including the case of pure bending, mainly due to the very low level of confinement of this type of tube in the first place, in comparison to Type I. This is evident by the fact that both the confined and unconfined models provided very similar predictions at high axial load level for Type II specimens. For this reason, there was no attempt to use the variable confinement model in Type II specimens, as the predicted interaction curve would have been very close to that predicted using the full confinement model. The full confinement model underestimated the moment at the balanced point by 4.6 percent and over estimated the axial load by 7.4 percent for Type II specimens.

By examining the interaction diagrams, four distinct zones can be recognized as shown in Fig. 5 (a) and (b). Zone 1 represents the contribution of plain concrete core alone. Zone 2 reflects the contribution of the FRP tube, provided that the confinement effect on the concrete core is ignored. In this case, it can be noticed that the contribution of the tube (zone 2) to the bending moment is much more significant than it is to the axial load, since it provides the flexural tension reinforcement element to the system. It should also be noted that the sizes of zones 1 and 2 are very similar for both Type I and II specimens. Zone 3 reflects the effect of the confinement mechanism imposed by the FRP tube, which is much more significant in case of Type I specimens than it is in Type II, due to the better laminate structure of the tube. The confinement effect is insignificant under pure bending, however, the contribution of zone 3 becomes more significant as the axial load increases, due to the larger portion of the section becoming under compression, as the neutral axis shifts. Zone 4 reflects the effect of the extended strain softening (ductility) for concrete beyond the 0.003 axial strain.

- **Load-deflection behavior of beams**

Fig. 3 shows the measured and predicted load-deflection diagrams of beams B1-I and B1-II. The prediction is based on the stress-strain curve of unconfined concrete with extended strain softening. Tension stiffening effect has also been considered (Fam 2000). The section analysis using layer-by-layer approach is performed for the case of pure bending and the moment-curvature response of the section is obtained. The deflection at mid span is calculated by integrating the curvatures along the span using the moment-area method. The model showed good agreement with the measured response, where the difference between the predicted and measured moment capacities were 1.82 and 8 percent for B1-I and B1-II respectively.

- **Load-axial strain behavior of concentrically loaded columns**

Fig. 4 shows the measured and predicted axial load-axial strain behavior of the columns of Type I and II specimens. The response was predicted using the stress-strain curve of the confined concrete (given in Fig. 9 for the case of zero eccentricity), which is based on the model by Fam and Rizkalla (2001 a). Mirmiran et al (1998) have shown that for length-to-diameter ratios (*L*/*D*) higher than 2:1, the strength of the confined concrete, f'_{cu}, is slightly reduced according to Equation 5.

$$\frac{f'_{cu}}{f'_{cu\,2:1}} = 0.0288\left(\frac{L}{D}\right)^2 - 0.263\left(\frac{L}{D}\right) + 1.418 \tag{5}$$

Where $f'_{cu} = f'_{cc}$ if the strength at ultimate is the peak value, while $f'_{cu} < f'_{cc}$ when the strength at ultimate is less than the peak strength in the case of low confinement effect such as in C1-II column. For a 3:1 ratio, the reduction is 12 percent, which has been accounted for. Due to the brittle nature of the relatively high strength concrete, the load-strain curves do not show a smooth response. Instead, a number of load drops are observed. For the C1-I column, the predicted response shows a reasonable agreement with the observed behavior. For the C1-II column the model over estimated the post-peak response of the strain-softening portion. The difference between the measured and predicted axial load capacities of both C1-I and C1-II was 11 percent.

Parametric Study

A parametric study has been conducted using the proposed analytical model in order to study the effects of the thickness of the FRP tube as well as different ratios of stiffness in the axial and hoop directions, which are controlled by the laminate structure. The parametric study considered a GFRP tube of 300 mm inner diameter and $[0/90]_s$ symmetric cross ply, E-glass/epoxy laminate, filled with 40 MPa concrete. Three different laminate structures of the tube are considered by varying the proportions of fibers in the axial [0] and hoop [90] directions, including 1:9, 1:1 and 9:1 ratios. A 1:9 laminate has 90 percent of the fibers oriented in the hoop direction, which is selected to provide high level of confinement. A 9:1 laminate has 90 percent of the fibers oriented in the axial direction and is selected to provide high flexural capacity and less confinement level for axial strength. A 1:1 laminate represents a balanced efficiency of the tube under bending and axial load. For each laminate, three values of wall thickness, *t*, have been considered, including 2, 8 and 16 mm, which are equivalent to reinforcement ratios of 2.7, 10.4 and 20.3 percent respectively. The variable confinement model of concrete was adopted in the analytical model to establish the interaction diagrams for the 9 cases under consideration. Fig. 10 shows the interaction diagrams obtained using the proposed variable confinement model, including three curves representing the three laminate structures for each tube's specific wall thickness. The axial load and bending moments are normalized with respect to the diameter D_o and concrete compressive strength f_c'. The figure clearly demonstrates the increased axial and bending capacities as the thickness of the tube is increased, as evident by the enlarged size of the interaction diagram for a specific laminate structure. Fig. 10 also demonstrates that increasing the axial stiffness of the tube (1:9 to 9:1) increases the pure flexural capacity significantly, for all range of wall thickness. On the other hand, the increase in pure axial strength might not necessarily be attributed to the increase of the hoop stiffness of the tube (9:1 to 1:9) for all the range of wall thickness. For example, in the case of 2 mm thick tube, the pure axial strength is increased as the hoop stiffness of the tube is increased, while in the case of 16 mm thick tube, the pure axial load has increased when the hoop stiffness of the tube was reduced. This interesting behavior can be explained by examining Equation 6, which represents the total ultimate axial load P_n in terms of the contributions of the concrete core, P_c, and the FRP tube, P_f (Fam 2000). P_c is the product of the confined strength of concrete f_{cc}' and the area of the concrete core, A_c, while P_f is the product of the stiffness of the tube in the axial direction in terms of the effective axial elastic modulus, E_f, the cross sectional area of the tube A_f, and the ultimate axial strain, ε_{cc}', which is assumed equal for both the tube and concrete core based on composite action.

$$\begin{aligned} P_n &= P_c + P_f \\ &= A_c f_{cc}' + A_f E_f \varepsilon_{cc}' \end{aligned} \tag{6}$$

As the hoop stiffness of the tube is reduced (1:9 to 9:1 for example), f_{cc}' is also reduced, and consequently, the first term of Equation 6 is also reduced. However, the second term of Equation 4 is increased simultaneously due to the increased axial stiffness of the tube E_f, on the expense of the reduced hoop stiffness (for the same wall thickness). For a small wall thickness, the rate of reduction of P_c is higher than the rate of increase of P_f when the hoop stiffness is reduced, and therefore, the over all ultimate load P_n is reduced. On the other hand, large thickness tubes would experience an over all increase in the ultimate axial load P_n despite of the reduction of the hoop stiffness, mainly because of the rate of increase of P_f, which is higher than the rate of reduction of P_c. At a certain intermediate wall thickness, the increase of P_f almost balances the reduction of P_c, and P_n remains almost constant, as shown in zone "A" of Fig. 10 for the 8 mm thick tube (D_o/t = 40). For tubes with larger wall thickness (D_o/t = 40 to 21), the interaction curves for different laminates do not intersect each other, while for smaller wall thickness (D_o/t = 40 to 152), the interaction curves intersect at certain points. This behaviour could have a great impact on optimisation from design point of view. For example, for tubes with small wall thickness, under small eccentricities such as e_1, shown in Fig. 10, the 1:9 laminate is the most efficient design out of all laminate structures, since it governs the outer envelope of the family of curves at this region, while under large eccentricities such as e_2, the 9:1 laminate is the most efficient design as it forms the outer envelope for all the curves. At certain intermediate eccentricities such as e_3, all laminates provide almost same efficiency, where the interaction curves intersect. On the other hand, for tubes with large thickness, the design efficiency for all range of eccentricities seem to be governed by tubes with highest axial stiffness such as the (9:1) laminate. It is also evident from Fig. 10 that a certain design strength (combination of *M* and *N*) can be achieved by several combinations of both laminate structure and wall thickness.

Summary and Conclusions

The axial load – bending moment interaction diagrams of concrete-filled FRP tubes, of two different types, have been established experimentally and analytically. The two types of tubes have almost the same diameter and wall thickness, however, the laminate structure of Type I tube resulted in much better confinement efficiency due to the lower Poisson's ratio and higher hoop stiffness. Three confinement mechanisms of concrete are examined, including an upper bound full confinement model, independent of the eccentricity of the load, a lower bound unconfined model, and a variable confinement model, accounting for the gradual change of state of bi-axial stresses developed in the tube as the eccentricity changes. Lamination theory, using ultimate laminate failure approach, was adopted to reflect the gradual reduction of stiffness of the tube due to the laminate progressive failure. Analytical model based on section analysis using layer-by-layer approach was developed. The model has been verified and extended to a parametric study to examine the effects of wall thickness and laminate structure of the tube using various proportions of fibers oriented in the axial and circumferential directions. The following conclusions are drawn:

1. Interaction curves of concrete-filled FRP tubes of moderate diameter-to-thickness ratios are similar to that of reinforced concrete members. As axial load increases, the moment capacity also increases and failure is governed by rupture of the FRP tube at the tension face. A balanced point is reached, beyond which, the moment capacity is reduced by increasing the axial load and failure is governed by crushing of the FRP tube in the compression side.
2. The variable confinement model of concrete provides best prediction of interaction diagrams. Full confinement model also provides reasonable prediction, however, for

tubes with adequate confinement stiffness (such as Type I), it overestimates bending capacity at low axial loads.

3. The unconfined concrete model significantly underestimates the interaction diagrams for concrete-filled FRP tubes with adequate confinement stiffness. However, the unconfined model with extended strain softening predicts very well the pure bending strength.
4. Laminate structure of FRP tubes significantly affects the interaction diagram. Type I tubes had higher effective elastic modulus in the hoop direction and significantly lower Poisson's ratio than Type II tubes, which resulted in better confinement as evident from the larger size of interaction curve and load - strain behavior of the columns of Type I.
5. For a given laminate structure, increasing the wall thickness of the tube increases the axial and bending strengths as evident from the increased size of interaction diagram. The curve could however change from the typical shape with the balanced point being the maximum moment (for small thickness tubes), to a shape with the pure bending strength being the maximum moment, and the full curve is governed by compression failure (for large thickness tubes, particularly for tubes with higher axial stiffness).
6. For both concrete-filled thin and thick tubes, increasing the ratio of fibers in axial direction significantly increases the flexural strength.
7. Increasing the ratio of fibers in the hoop direction would increase the axial strength of concrete-filled thin tubes only.
8. Axial strength of concrete-filled thick tubes tends to increase by increasing the amount of fibers in the axial direction rather than in the hoop direction. In thick tubes, the contribution from axial stiffness of the tube is more significant than the gain from confinement.

9. For small thickness tubes, changing the proportion of fibers in the axial and hoop directions, results in a family of interaction curves, intersecting at certain points, which provides an optimum laminate structure for any particular eccentricity, for a given wall thickness. For relatively thick tubes, the interaction curves do not intersect and the optimum laminate seems to be the one with maximum axial stiffness and minimum hoop stiffness, regardless of the eccentricity.
10. There are several combinations of wall thickness and laminate structure that satisfies a particular combination of bending moment and axial load.

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Notations

A_c = Cross-sectional area of the concrete core

A_f = Cross-sectional area of the FRP tube

$CC(i)$ = Compression force in concrete in a general strip *i* in a concrete-filled FRP tube

$CF(i)$ = Compression force in FRP within a general strip *i* in a concrete-filled FRP tube

c = Neutral axis depth

D = Diameter of concrete-filled FRP tube

D_o = Outer diameter of the tube

E_{co} = Initial tangential elastic modulus of concrete

E_{sec} = Secant modulus of concrete at $f_c^{'}$

E_f = Effective elastic modulus of FRP tube in the axial direction

e = Eccentricity of axial load measured to the center of the circle

f_c = Axial compressive stress in concrete

$f_c(i)$ = Axial stress level in concrete at a general strip *i* in a concrete-filled FRP tube

$f_c^{'}$ = Concrete compressive strength

$f_{cc}^{'}$ = Peak axial strength of FRP confined concrete under concentric axial load

f_{cco} = Axial compressive stress in concrete corresponding to ε_{cco}

$f_{cu}^{'}$ = Compressive strength of FRP confined concrete at ultimate

$\overline{f_{cc}}$ = Peak axial compressive strength of partially confined concrete in presence of eccentricity

$f_f(i)$ = Axial stress level in the FRP tube at a general strip *i* in a concrete-filled FRP tube

H = Height of concentrically or eccentrically loaded column specimens

i = General strip within the cross-section of a concrete-filled FRP tube

L = Span of beams

L = Length of column

M = Bending moment

M_n = Flexural capacity

N = Axial compression force

P_c = Load carried by concrete in concrete-filled tube under axial compression load

P_f	=	Load carried by the FRP tube in concrete-filled tube under axial compression load
P_n	=	Axial compression capacity
r	=	Constant in Mander's equation relates the initial tangential modulus E_{co} to the secant modulus of concrete E_{sec}
$TF(i)$	=	Tension force in the FRP tube in a general strip i in a concrete-filled FRP tube
t	=	Wall thickness of FRP tube
α	=	Shape parameter for the stress-strain curve of partially confined concrete
$\varepsilon(i)$	=	Axial strain at a general strip i within the cross-section of concrete-filled FRP tube
ε_c	=	Axial compressive strain corresponding to f_c
ε_c	=	Compressive strain at the extreme concrete fibers of the member
ε_{cco}	=	Ultimate axial compressive strain of concrete, which equals to the ultimate axial strain of the FRP tube in compression.
$\varepsilon_c^{'}$	=	Axial compressive strain of concrete corresponding to
$\varepsilon_{cc}^{'}$	=	Axial strain of concrete corresponding to $f_{cc}^{'}$
$\overline{\varepsilon_{cc}}$	=	Axial compressive strain corresponding to $\overline{f_{cc}}$
x	=	Ratio between any axial strain level ε_c and the strain $\varepsilon_c^{'}$
$\overline{x}$	=	The ratio $\overline{\varepsilon_{cc}} / \varepsilon_c^{'}$

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