DEFORMABILITY OF FLEXURAL CONCRETE MEMBERS PRESTRESSED WITH FRP

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ABSTRACT: A parametric study is conducted to evaluate the deformability of rectangular and T-sections prestressed by carbon fibre reinforced polymers, CFRP. Deformability of beams prestressed by CFRP is compared to that of beams with steel. A unified design approach for concrete sections with FRP is concluded based on minimum and maximum values of the reinforcement ratio. The energy dissipated before failure of sections prestressed by CFRP is compared to that of sections prestressed by steel. Recommendations and design guidelines based on deformability are presented.

KEYWORDS: beam, carbon, deformability, design, ductility, energy, fibre reinforced polymers, prestressed concrete.

1 INTRODUCTION

Use of concrete members reinforced or prestressed by fibre reinforced polymers, FRP, reinforcement has increased recently to solve the traditional problem of corrosion of steel reinforcement. The linear perfectly elastic characteristics of FRP without the traditional ductility of steel reinforcement raise the concern on the degree of ductility and consequently the safety of members prestressed with FRP. Test results of flexural members prestressed by FRP show ample warning before failure in the form of excessive deflection and large crack width, therefore, there is an adequate safety of such members.

Ductility of concrete structures is one of the prime consideration in the design of concrete structures. For concrete structures prestressed by steel, ductility is defined by the ability of the member to sustain inelastic deformations without loss in its load carrying capacity. Since FRP behaves linearly elastic up to failure, failure of flexural members prestressed with FRP reinforcement could occur due to either crushing of concrete or rupture of the tensile reinforcement which are both brittle. Due to this fundamental difference, the traditional definition of ductility based on deformation of the member at yielding of the prestressing reinforcement can not be applied for beams prestressed with FRP.

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Members prestressed with FRP should be designed to achieve two main criteria, significant deformations and extensive cracks before failure to provide ample warning and consequently safety of the structure. Members should also have adequate energy to absorb any excessive loading beyond the service loads. Since the absorbed energy of concrete beams prestressed with FRP is mainly elastic, failure of these members is expected to be severe and brittle.

2 FLEXURAL BEHAVIOUR

Concrete beams prestressed to the same level of stress using FRP and steel behave identically before cracking. After cracking, beams prestressed with FRP exhibit more deformations and larger crack widths than that of beams with steel due to the lower elastic modulus of FRP. Failure of beams with FRP is typically governed by either crushing of concrete or rupture of FRP. After cracking, the depth of the compression zone of T-sections prestressed by FRP becomes significantly smaller than that of rectangular sections. Consequently, for T-sections, the tensile strain in the reinforcement reaches its ultimate value before the concrete fails and failure usually occurs due to rupture of FRP reinforcement. On the other hand, for rectangular sections, the tensile strain in the reinforcement at failure could be less than its ultimate value and failure could be due to crushing of concrete or rupture of FRP depending on the reinforcement ratio provided.

As a typical example of beams prestressed by carbon FRP, CFRP, the compression zone depth, \( c \), of two beams with rectangular and T-sections is shown in Figure 1. Using a typical value of the prestressing level after losses, \( f_p \), as 1000 MPa, and the material properties of the CFRP and steel reinforcement given in Table 1, the balanced reinforcement ratios, \( \rho_{br} \), for rectangular and T-sections shown in Figure 1 are 0.005 and 0.023, respectively. The compression zone depth is more for rectangular than T-sections as shown in Figure 1.

Figure 1 Compression zone depth of beams prestressed by CFRP
Table 1 Material properties of prestressing reinforcement

<table>
<thead>
<tr>
<th>Material</th>
<th>ultimate strength, $f_u$ (MPa)</th>
<th>Elastic modulus (GPa)</th>
<th>ultimate strain, $\varepsilon_u$</th>
<th>Stress after losses, $f_e$ (MPa)</th>
<th>$f_e / f_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFRP</td>
<td>2000</td>
<td>147</td>
<td>0.0136</td>
<td>1000</td>
<td>0.500</td>
</tr>
<tr>
<td>Steel</td>
<td>1860</td>
<td>190</td>
<td>0.04</td>
<td>1000</td>
<td>0.538</td>
</tr>
</tbody>
</table>

Typical moment-curvature relationships of sections prestressed by CFRP and steel are shown in Figures 2 and 3 for rectangular and T-sections respectively. For rectangular sections, the curvature at ultimate of both sections prestressed with CFRP and steel are matching, however, the curvature for T-sections is much less for section with CFRP than that with steel. For the rectangular sections shown in Figure 2, the failure is governed by crushing of concrete and the behaviour of the beams after cracking is non-linear due to the non-linear stress-strain characteristic of the concrete. However for T-sections, the failure of the beam with CFRP is due to rupture of the reinforcement and the strain in the concrete before failure is relatively small leading to linear behaviour after cracking as shown in Figure 3.

3 DEFORMABILITY

Several definitions have been proposed to quantify the deformability of beams with FRP [1,2,3]. The definition given by Abdelrahman et al. [1] can be applied for any beam reinforced or prestressed by FRP. Jaeger et al. [2] proposed a definition for rectangular beams reinforced by FRP based on the moment and curvature at ultimate, $M_{u}$, $\phi_{u}$ and the moment and curvature at concrete strain of 0.001, $M_{(0.001)p}$, $\phi_{(0.001)p}$ respectively, as given by eq.1. It has been reported that the overall factor is the same for beams reinforced by FRP and steel [2]. This is attributed to the higher strength of FRP than steel and the higher reinforcement ratio required for beams.
reinforced with FRP to satisfy the serviceability requirement. Therefore, the moment capacity of beams reinforced by FRP is typically higher than that for beams with steel and consequently the overall factor for both types of beams is the same. The overall factor is calculated for T-sections prestressed by CFRP and steel as shown in Figure 4. Since the ultimate strength of CFRP is similar to that of prestressing steel strands for the same percentage of prestressing reinforcement, the moment capacity is also the same for both types of beams. Therefore, the overall factor for beams prestressed by steel is much higher than that with CFRP as shown in Figure 4. It can be seen from Figure 4 that for specific configuration of T-section, beams prestressed by steel with reinforcement ratio of 1.0 percent requires 2 to 3 times of CFRP reinforcement to achieve the same overall factor. For reinforcement ratios between 1 and 2.5 percent, the overall factor for beams prestressed by CFRP is much less than that for beams prestressed by steel. Similar overall factor cannot be achieved by increasing the CFRP reinforcement ratio. For higher reinforcement ratios, the overall factor is identical, since the failure is controlled by crushing of concrete regardless of the type of reinforcement.

\[
\text{overall factor} = \frac{M_u \varphi_u}{M_{(0.001)} \varphi_{(0.001)}}
\]

(1)

![Graph of overall factor for Prestressed T-sections with CFRP](image)

Figure 4 Overall factor for Prestressed T-sections with CFRP

Naaman et al. [3] defined the ductility of beams prestressed by FRP using the unloading path of the load-deflection relationship of concrete beams. The proposed ductility index given by eq.2 was applied for a group of beams prestressed with CFRP tested at the University of Manitoba. The beams had a T-section and a span of 5.8 metre [1,4]. The calculated ductility index for one of the beams based on the measured deflection at unloading prior to failure was 1.23 compared to a calculated value of 3.00 based on the predicted unloading curve proposed by Naaman et al [3] as shown in Figure 5.

\[
\mu = \frac{1}{2} \left( \frac{E_{tot}}{E_{el}} + 1 \right)
\]

(2)
The ACI Code 318/318R-95 [5] introduces a unified design procedure for reinforced and prestressed concrete flexural and compression members using the net tensile strain approach. The net tensile strain is defined for prestressed sections as the strain in the tension reinforcement after decompression. The ACI Code specifies that when the net tensile strain in the extreme tension reinforcement at nominal strength of the section is greater than 0.005, the section is defined as tension-controlled where ample warning of failure with extensive deflection and cracking are expected. Adopting this approach for beams prestressed by CFRP will lead to maximum reinforcement ratio which can be used by designers to provide sufficient deformability.

Results of two recent studies conducted at the University of Manitoba [4,6] on 13 concrete beams, 6.2 to 9.3 metre long, prestressed by different types of CFRP reinforcement indicated that the deflection is in the range of 1/70 to 1/110, where I is the span of the beam, at net tensile strain of 0.005. The measured crack width was ranging from 0.8 to 1.4 mm at the same net tensile strain value. This is certainly an excessive deflection and crack width and are sufficient to provide ample warning before failure.

Failure mode of beams prestressed by CFRP with the maximum reinforcement ratio is typically due to crushing of concrete. Therefore, the strain distribution at the time of failure can be determined based on the ultimate compressive strain of the concrete, 0.0035, and the strain at the level of the prestressing reinforcement of 0.005. Therefore, the value of (c/d) ratio at ultimate can be limited to a value of 0.4 as given by eq.3.

$$\left(\frac{c}{d}\right)_{\text{max}} = 0.4$$  \hspace{1cm} (3)

Using this approach, the maximum reinforcement ratio is shown in Figure 6 for rectangular and T-sections with different flange widths at an effective prestress level of 1000 MPa. For rectangular sections, the maximum reinforcement ratio is calculated to be 0.007. The maximum reinforcement ratio increases by increasing the dimensions of the compression flange as shown in Figure 6.
The limit of the maximum (c/d) ratio at ultimate may be waived if the moment of resistance, $M_0$, is greater than 1.5 times the factored moment, $M$. The factor of 1.5 has been selected conservatively due to lack of research on statistical evaluation of the safety index currently used in the code using FRP material characteristics and load factors.

![Graph showing the limit of the maximum (c/d) ratio at ultimate.](image)

Figure 6 Maximum reinforcement ratio of beams prestressed by CFRP

Due to the brittle nature of the failure of beams with FRP reinforcement, a minimum reinforcement ratio should also be specified as proposed by eq.4, where $M_r$ and $M_c$ are the moment of resistance and the cracking moment of sections prestressed by FRP, respectively. Using material factors for concrete, ($\phi_c$), and CFRP, ($\phi_p$), of 0.75 and 0.81, respectively, the ratio of the nominal strength, $M_0$, to the moment of resistance, $M_r$, is ranging between 1.2 and 1.3 for beams prestressed by CFRP. Therefore, the minimum reinforcement ratio could be specified according to eq.5. Based on eq.5, the minimum reinforcement ratio is shown for rectangular and T-sections with different flange widths in Figure 7.

![Graph showing the minimum reinforcement ratio of beams prestressed by CFRP.](image)

Figure 7 Minimum reinforcement ratio of beams prestressed by CFRP
\[ M_r \geq 1.2 \, M_{cr} \]  \hspace{2cm} (4)
\[ M_n \geq 1.5 \, M_{cr} \]  \hspace{2cm} (5)

4 ENERGY DISSIPATION

The energy consumed by the structural system before failure is typically used as a measurement of the ductility. Ductility of structures is needed to sustain any unexpected overloading conditions. Behaviour of concrete beams prestressed by CFRP reinforcement is non-linearly elastic up to failure, where the non-linearity arises from cracking of the concrete [4]. The inelastic energy consumed prior to failure of beams prestressed by CFRP is very small in comparison to beams prestressed by steel provided that the prestressing steel is allowed to yield before failure. Conversely, the elastic energy released at failure of beams prestressed by CFRP is tremendous similar to the brittle shear failure of prestressed beams.

The total energy, including elastic and non-elastic energy, of a prestressed beam can be calculated as the area under the load-deflection or the moment-curvature diagrams. The energy dissipated before failure of beams prestressed by CFRP is compared to that of beams prestressed by steel strands with the same prestressing force in Figures 8 and 9. For rectangular sections, the energy of beams prestressed by CFRP is less than that of beams with steel at low reinforcement ratio where the failure is governed by rupture of reinforcement as shown in Figure 8. At balanced reinforcement ratio, \( \rho_{br} \), where the mode of failure changes to crushing of concrete, there is virtually no difference in the energy consumed by both types of beams. The energy of T-sections prestressed by CFRP is much less than that of beams prestressed by steel as shown in Figure 9. The maximum energy obtained from a T-section prestressed by CFRP is at a reinforcement ratio equal to \( \rho_{br} \). The energy of beams prestressed by steel is also calculated at net tensile strain of 0.005 and 0.0075 as shown in Figures 8 and 9. It can be seen that the energy of beams prestressed by CFRP matches the energy of beams with steel at net tensile strain of 0.0075.

Figure 8 Energy of prestressed rectangular beams  Figure 9 Energy of prestressed T-beams
The ACI Code [5] allows for a net tensile strain of 0.005 to ensure ductile behaviour and 0.0075 net tensile strain to allow for redistribution of moments in continuous members. Redistribution of moments in members prestressed with CFRP can only be achieved due to cracking of the concrete, therefore, it is considered less than that for members prestressed by steel reinforcement [7].

5 CONCLUSIONS

This paper introduces design considerations for beams prestressed by CFRP based on deformability. A minimum reinforcement ratio is proposed based on achieving a nominal strength of at least 2.0 times the cracking moment of the section. A maximum limit for the reinforcement ratio is proposed based on a net tensile strain in the reinforcement at nominal strength of the section of 0.005. At this strain level, the deflection and the crack width of beams prestressed by CFRP are considerably large. The maximum limit of (c/d) ratio is calculated to be 0.4, however this limit may be waived if the moment of resistance is greater than 1.5 times the factored moment.

The energy dissipated of rectangular sections prestressed by CFRP failing by crushing of concrete matches that of beams prestressed by steel. However, the energy dissipated before failure of T-sections prestressed by CFRP is much smaller than that of sections with steel strands. The energy of beams prestressed by CFRP matches the energy of beams prestressed by steel calculated at net tensile strain of 0.0075.

REFERENCES