The Prediction of Air Leakage Rates Through Cracks in Pressurized Reinforced Concrete Containment Vessels

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ABSTRACT

Measurements of air leakage through cracks in specimens of reinforced concrete were analysed on the basis that the compressible flow occurred without heat transfer between the air and the concrete. The resultant friction coefficients were then correlated with the crack size and roughness parameters.

The empirical relations were rearranged so that leakage rates could be predicted for a gas with known properties passing through cracks of known average dimensions. Checking the predicted leakage against the measured leakage rates gave agreement within ±35%, the scatter being attributable to the very irregular nature of cracks. The method presented for the prediction of leakage can be used with confidence for typical cracks in reinforced concrete containment vessels.

NOTATION

\[ a \] Coefficient in eqn (5)

\[ \bar{a} \] Average of the \( a \)-coefficients

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INTRODUCTION

If excessive pressures build up inside an unlined containment structure made of reinforced concrete, the tensile stresses may result in cracks through which the fluid will leak. When the fluid is a gas, its compressibility under such high pressure complicates the problem of predicting leakage rates.

A series of tests was made by Lau\(^1\) to measure the leakage of air through rectangular specimens of reinforced concrete loaded in tension to produce cracks. The results were analysed in Lau’s thesis and empirical factors developed to form relations for predicting leakage rates. This work was extended by Rizkalla, Lau and Simmonds\(^2\) to combine the prediction of cracks and leakage through them.
The present report represents a re-analysis of the leakage test results\(^1\) by making a more complete allowance for compressibility and roughness, with the objective of developing a reliable method for predicting leakage rates from cracked containment vessels.

**LEAKAGE RATE MEASUREMENTS**

Lau\(^1\) gave full details of the test apparatus, experimental techniques and measurements; a summarized version, more readily available, has been presented by Rizkalla et al.\(^2\) Briefly, rectangular specimens of reinforced concrete 760 mm long by 300 mm wide, and of three thicknesses (127, 170 and 254 mm) were loaded longitudinally in tension to produce cracks. When cracks appeared they were counted and measured. Then the flow of air through the cracks was measured when a pressure differential was applied across the specimen at 20 kPa increments up to 200 kPa (gauge). The cracking was increased by increasing the load on the concrete in steps up to a maximum of 400 MPa, and the crack and flow measurements were repeated at each step.

**ANALYSIS OF EXPERIMENTAL RESULTS**

Under the maximum air pressure loading, the air expanded from approximately 300 kPa (abs) to 100 kPa (abs). Clearly the effects of compressibility are significant. In the earlier analyses\(^1,2\) it was assumed that the thermodynamic expansion process was isothermal. In the present re-analysis, the process is assumed to be adiabatic, i.e. there is no heat transfer to the air from the concrete.

The difference between adiabatic and isothermal processes becomes more marked as the pressure ratio increases. The real process is probably somewhere between the two, but will be nearer to the adiabatic process because the heat transfer is very low because gases have poor thermal conductivity, the gas velocity is very low, and the temperature difference between the concrete and gas is small. It might be thought that the isothermal process would automatically give zero temperature difference and therefore zero heat transfer, but it must be remembered that the isothermal expansion of a gas implies that, during the process, heat is added to the expanding gas from external sources.

The adiabatic flow of a compressible gas through a constant-area duct with friction is usually referred to as Fanno flow in gas-dynamic books such as Shapiro.\(^3\) Fanno flow assumes that the friction coefficient of the duct is
known. In practice, the friction coefficient depends in some manner upon the fluid properties and duct geometry. For the present purposes, the friction coefficient will be calculated from the experimentally measured flow rates. Then, dimensional analysis will be used to deduce the most significant crack parameters for correlation with the experimental crack variables.

The relations which govern the leaking gas flow have been developed in Appendix 1; the basic ideas are repeated here. The gas (with gas constant, $R$, and ratio of specific heats, $k$) is assumed to flow isentropically from the inside of the pressurized vessel, where the temperature and pressure are $T_o$ and $p_{o,i}$, to the inlet of the crack. Within the crack, the adiabatic flow is resisted by the viscous forces manifested in the friction coefficient, $f$, assumed to be constant along the crack length, $L$. On leaving the crack the gas is assumed to decelerate isentropically to the pressure, $p_{o,e}$, external to the vessel. The resulting equations are:

$$G = p_{o,i}(k/RT_o)^{1/2}M_i\{1 + M_i^2(k - 1)/2\}^{-(k+1)/2(k-1)}$$  \hspace{1cm} (1)

$$G = p_{o,e}(k/RT_o)^{1/2}M_e\{1 + M_e^2(k - 1)/2\}^{-(k+1)/2(k-1)}$$  \hspace{1cm} (2)

and

$$f = \frac{W}{2kL} \left[ 1 - \frac{1}{M_i^2} + \frac{k + 1}{2} \ln \frac{M_i^2\{1 + M_i^2(k - 1)/2\}}{M_e^2\{1 + M_e^2(k - 1)/2\}} \right]$$  \hspace{1cm} (3)

where $G$ is the mass flow per unit area of crack, $M_i$ and $M_e$ are the inlet and exit Mach numbers just inside the crack, and $W$ is the average crack width. These equations are sufficient to determine the friction coefficient from the experimental measurements.

It is now necessary to relate the crack characteristics to the friction coefficient. This is done by using the functional relation (A13) from Appendix 2:

$$f = f(\mu/LG, W/L, \varepsilon/L, s)$$  \hspace{1cm} (4)

where $\mu$ is the absolute coefficient of viscosity, $\varepsilon$ is a measure of the roughness height, and $s$ is a measure of the shape of the crack. The shape parameter, $s$, is the most difficult to define and control: it takes into account the crack irregularities in the direction of, and transverse to, the flow. Fortunately, for the present purposes, it can be argued that the tests were done in reinforced concrete and the results are to be applied to cracks in reinforced concrete; therefore, the shape of the cracks will not change significantly from case to case and the effects of shape will be ignored.

Furthermore, the tests were performed on specimens of the same concrete mix (so that $\varepsilon$ was effectively constant) but with three values of $L$ so that the tests covered three values of $\varepsilon/L$. Also, the tests were done such that the
concrete load (and hence $W/L$) was held constant while the flow rate was varied. Therefore, the tests allow the effects of each of the parameters to be observed in turn.

Only seven of the eight specimens tested gave useful results and these have been analysed using the above procedures.

**Evaluation of friction coefficient, $f$**

For each flow measurement, the values of $G$, $p_{e,i}$, $T_o$ and $p_{o,e}$ were used in eqns (1) and (2) to calculate $M_i$ and $M_e$ using the Newton–Raphson method. Then eqn (3) was used with the crack dimensions $W$ and $L$ to determine $f$.

**Dependence of $f$ on $\mu/LG$**

The values of $f$ were plotted against $\mu/LG$ for each set of measurements in which the air pressure differential was increased while the crack width was held constant. A typical result is shown in Fig. 1. All the curves were linear, indicating that

$$f = a + b(\mu/LG)$$

where the coefficients $a$ and $b$, in general, vary with $W$ and $\varepsilon$, and have been evaluated.

![Graph showing variation of friction coefficient with flow parameter.](image-url)
Dependence of $a$ and $b$ on $W/L$

With the assumption that the roughness, $\varepsilon$, is constant for all the concrete specimens, the three model sizes of $L$ gave three values of the parameter $\varepsilon/L$. The coefficients $a$ and $b$ were plotted against $W/L$ (as in Figs 2 and 3) for each of the three $\varepsilon/L$ values. The values of $a$ and $b$ do not vary very much with $W/L$ and certainly not in any regular manner; therefore, it was decided to treat $a$ and $b$ as constants (for constant $\varepsilon/L$), the constants being the arithmetic means denoted by $\bar{a}$ and $\bar{b}$.

Fig. 2. Dependence of coefficient $a$ on crack width.

Fig. 3. Dependence of coefficient $b$ on crack width.
Dependence of $\tilde{a}$ and $\tilde{b}$ on $\varepsilon/L$

The mean diameter of the sand grains (0.76 mm) was used as a measure of the roughness and then, with the three test values of $\varepsilon/L$, $\tilde{a}$, and $\tilde{b}$ were expressed as three-term polynomials of $\varepsilon/L$ to give:

\[
\tilde{a} = 28.1 - 10.5 \times 10^3(\varepsilon/L) + 0.987 \times 10^6(\varepsilon/L)^2
\]

and

\[
\tilde{b} = 677 000 - 223 000 \times 10^3(\varepsilon/L) + 20 000 \times 10^6(\varepsilon/L)^2
\]

General expression for $f$

Combining the results of the previous sections, the function for the friction coefficient, $f$, as given by eqn (4), becomes:

\[
f = 28.1 - 10.5 \times 10^3(\varepsilon/L) + 0.987 \times 10^6(\varepsilon/L)^2
\]

\[
+ \{677 000 - 223 000 \times 10^3(\varepsilon/L) + 20 000 \times 10^6(\varepsilon/L)^2\}(\mu/GL)
\]

PREDICTION METHOD

Principle of prediction method

To predict the gas leakage it is required to know the gas characteristics $k$, $R$, $\mu$, $T_o$, $p_{o,l}$, $p_{o,e}$, and the concrete and crack characteristics $L$, $W$, $B$ (the lateral extent of the crack—the total external length of crack on the inlet face neglecting waviness of the crack line) and $\varepsilon$. These variables are inserted into eqns (1), (2) (3) and (8), which now define $G$, $M_p$, $M_e$ and $f$ and allow $G$ to be determined by a method such as the one given in Appendix 3. The actual mass flow rate will then be GBW.

Prediction of leakage through the experimental models

The concrete specimens used in the experiment were treated as parts of a reinforced concrete containment vessel and the mass flow rates predicted using the method outlined above. The predicted leakage is compared with the measured leakage in Fig. 4. Here, the mass flow is presented non-dimensionally with respect to $T_o$ and $p_{o,e}$ at the exit because, in the experiment, it was the exit volume flow which was measured. The agreement, generally better than $\pm 35\%$, is excellent considering the irregular nature of cracks. This gives confidence in applying the method to practical structures.
Fig. 4. Leakage prediction method applied to the experimental models.

**Prediction of leakage through cracked reinforced concrete containment vessels**

The shell of a containment vessel will not be a uniform membrane, neither will a crack pattern develop uniformly; therefore, the leakage rate will vary over the vessel surface. It can be argued that the resistance to flow through the cracks is large and so the amount of flow will have very little effect on the internal and external pressures and the cracks behave as a set of independent flow paths in parallel. It is therefore possible to subdivide the vessel surface into local areas with similar crack characteristics, to calculate the leakage through each local area and then, to sum the leakages. In the summing, it must be remembered that the mass flow quantity, $G$, is in terms of mass flow per unit area of crack and the total mass flow will be given by

$$\text{total mass flow} = \sum G_j B_j W_j$$

where the suffix $j$ is the identifier of a subdivision of the vessel shell.

The application of this method to lined containment structures is outside
the scope of this paper, but some of the principles may be noted. A lining fracture forms an orifice in series with one or more of the concrete cracks and the analysis will follow that of pipe networks. If the lining fracture is large, its pressure loss will be relatively small and its flow characteristics will be as for an incompressible flow orifice. For smaller lining fractures, the effects of compressibility becomes more and more pronounced, until the pressure ratio across the lining fracture is sufficient to produce the choking phenomenon and the leakage becomes independent of the downstream concrete cracks.

CONCLUSIONS

Experimental results for the leakage of pressurized air through cracks in reinforced concrete blocks have been analysed on the basis that the thermodynamic process through the cracks may be considered to be Fanno flow.

It was found that the friction coefficient was linearly related to the inverse of the 'crack Reynolds number' so that

\[ f = a + b(\mu/LG) \]

The coefficients \(a\) and \(b\), were approximately independent of the crack width-to-length ratio and could be represented by the arithmetic means at constant roughness \(\tilde{a}\) and \(\tilde{b}\), which were dependent on the roughness. By taking the size of the sand in the concrete as a measure of the roughness, simple empirical relations were determined for the effects of roughness:

\[ \tilde{a} = 28.1 - 10.5 \times 10^3(e/L) + 0.987 \times 10^6(e/L)^2 \]
\[ \tilde{b} = 677,000 - 223,000 \times 10^3(e/L) + 20,000 \times 10^6(e/L)^2 \]

Application of these relations to predict the experimental leakage rates gave agreement within \(\pm 35\%\), the scatter being attributed to the very irregular nature of cracks.

The relations may be used with reasonable confidence to predict the leakage rates through cracks in unlined reinforced concrete containment vessels.

REFERENCES


APPENDIX 1

Isentropic and Fanno flows

*Thermodynamic models*

The leakage of a pressurized gas through cracks in the walls of its containment vessel may be considered to be a series of idealized thermodynamic processes. First, the gas accelerates from its state of rest in the vessel to the crack inlet conditions. This acceleration is made isentropically, i.e. without any heat passing to or from the gas (adiabatically) and without friction. Secondly, the crack is considered to be a constant-area straight duct in which the flow is resisted by the gas’s viscosity, and the flow process is without heat transfer because of the gas’s low thermal conductivity, low velocity and small temperature difference with the solid. This constant-area, adiabatic flow with friction is known as Fanno flow. The third and final process is the isentropic deceleration of the flow from the crack exit conditions to atmospheric pressure and zero velocity.

The three flow regions are illustrated in Fig. 5(a) by inlet and exit reservoirs smoothly interconnected by the frictional duct.

*Isentropic flow*

The flow originates from the inlet reservoir with conditions denoted by $\left(\right)_{\text{in}}$ and accelerates to the ‘crack’ inlet where the conditions are denoted by $\left(\right)_i$. Temperature, pressure and density relations for isentropic flow are given by eqns (4.14) of Ref. 3 and, in the present notation, become:

$$T_{\text{o},i}/T_i = 1 + M_i^2(k - 1)/2 \quad (A1)$$

$$p_{\text{o},i}/p_i = \left\{1 + M_i^2(k - 1)/2\right\}^{[k/(k - 1)]} \quad (A2)$$

$$\rho_{\text{o},i}/\rho_i = \left\{1 + M_i^2(k - 1)/2\right\}^{1/(k - 1)} \quad (A3)$$

The mass flow per unit area is

$$G = \rho_i V_i \quad (A4)$$

which can be expressed in terms of the upstream reservoir pressure and inlet Mach number by successively using the equation of state,

$$\rho = p/RT \quad (A5)$$
The prediction of air leakage rates

(a) Fanno flow model

(b) Fictitious extension of duct to sonic condition

Fig. 5. Idealized adiabatic flow through crack.

the relations for the Mach number and speed of sound,

\[ M = \frac{V}{c} = \frac{V}{(kRT)^{1/2}} \quad (A6) \]

and the isentropic relations (A1) and (A2), so that

\[ G = p_{o,i} \left\{ \frac{k}{RT_{o,i}} \right\}^{1/2} M_{i} \left\{ 1 + M_{i}^{2} (k - 1)/2 \right\}^{-(k+1)/(k-1)} \quad (A7) \]

Now, since the mass flow per unit area must be the same at all stations along the constant-area duct, a similar expression can be written in terms of the exit and exit reservoir conditions, because the deceleration process is assumed to be isentropic. Then

\[ G = p_{o,e} \left\{ \frac{k}{RT_{o,e}} \right\}^{1/2} M_{e} \left\{ 1 + M_{e}^{2} (k - 1)/2 \right\}^{-(k+1)/(k-1)} \quad (A8) \]

A further simplification is possible because, for adiabatic (and isentropic) flow, the total temperature is constant so that

\[ T_{o,i} = T_{o,e} = T_{o} \quad (A9) \]

**Fanno flow**

The main assumptions of Fanno flow are that the flow is steady and one-dimensional, there is no heat transfer, the duct is of constant cross section, and the friction coefficient is constant along the length of the duct. Crack flow can be safely represented by Fanno flow provided that crack irregularities affect only the friction coefficient. The analysis is made more
convenient if the actual crack length, \( L \), is increased by a fictitious amount, \( \tilde{L}_e \), to give a total length, \( \tilde{L}_t \), as shown in Fig. 5(b). The lengthening is such that the extension terminates where the local flow is sonic. Under these conditions, eqn (6.20) of Ref. 3 gives

\[
\frac{2f\tilde{L}_t}{W} = \frac{1 - M^2}{kM^2} + \frac{k + 1}{2k} \ln \left\{ \frac{M^2(k + 1)}{2 + M^2(k - 1)} \right\} \tag{A10}
\]

where the hydraulic diameter of the crack has been taken as \( 2W \), appropriate for narrow rectangular ducts.

Application of the same relationship to the fictitious extension gives

\[
\frac{2f\tilde{L}_e}{W} = \frac{1 - M^2}{kM^2} + \frac{k + 1}{2k} \ln \left\{ \frac{M^2(k + 1)}{2 + M^2(k - 1)} \right\} \tag{A11}
\]

Now, the fictitious length of duct can be eliminated by subtracting eqn (A11) from eqn (A10). A slight rearrangement then gives

\[
f = \frac{W}{2Lk} \left[ \frac{1}{M^2} - \frac{1}{M^2} + \frac{k + 1}{2} \ln \left\{ \frac{M^2(2 + M^2(k - 1))}{M^2(2 + M^2(k - 1))} \right\} \right] \tag{A12}
\]

**APPENDIX 2**

**Roughness effects**

The gas flow through the crack is resisted by the shear stress, \( \tau \), between the gas and the solid. The magnitude of the shear stress will depend on the following variables defining the gas and the solid: gas density, \( \rho \), gas velocity, \( V \), gas absolute coefficient of viscosity, \( \mu \), average crack width, \( W \), height of crack surface roughness, \( \varepsilon \), shape of the crack, \( s \) (assumed to be non-dimensional). By using dimensional analysis, these variables can be grouped into the non-dimensional parameters

\[
\tau/\rho V^2, \quad \mu/\rho VL, \quad W/L, \quad \varepsilon/L, \quad s
\]

and, without loss of generality, \( \tau/\rho V^2 \) can be replaced by the average friction coefficient, \( f' \), since

\[
f = (1/L) \int_0^L (\tau/\rho V^2) dx
\]

and \( \rho V \) can be replaced by \( G \). The following functional relation can be written:

\[
f = f(\mu/LG, W/L, \varepsilon/L, s) \tag{A13}
\]
APPENDIX 3

Prediction of leakage rate

It is possible to avoid the explicit use of the friction coefficient by eliminating it between eqns (3) and (5) to give

\[
\frac{W}{2kL} \left[ \frac{1}{M^2_i} - \frac{1}{M^2_e} + \frac{k+1}{2} \ln \left\{ \frac{M^2_i \{1 + M^2_e(k-1)/2\}}{M^2_e \{1 + M^2_i(k-1)/2\}} \right\} \right] - (\bar{a} + \delta \mu/LG) = 0
\]

(A14)

where the coefficients \( \bar{a} \) and \( \delta \) are given by eqns (6) and (7). Then the variables \( G, M_i \) and \( M_e \) can be determined by simultaneously solving three non-linear equations, (1), (2) and (A14), using the Newton–Raphson method.

Approximate values of the variables satisfy the following relations:

\[
\frac{W}{2kL} \left[ \frac{1}{M^2_i} - \frac{1}{M^2_e} + \frac{k+1}{2} \ln \left\{ \frac{M^2_i \{1 + M^2_e(k-1)/2\}}{M^2_e \{1 + M^2_i(k-1)/2\}} \right\} \right] - (\bar{a} + \delta \mu/LG) = g_1
\]

(A15)

\[
M^2_i \{1 + M^2_e(k-1)/2\}^{-1/2(k-1)} - (G/p_{0,a})(RT_a/k)^{1/2} = g_2
\]

(A16)

and

\[
M^2_e \{1 + M^2_i(k-1)/2\}^{-1/2(k-1)} - (G/p_{0,a})(RT_a/k)^{1/2} = g_3
\]

(A17)

where \( g_1, g_2 \) and \( g_3 \) are corrections for the inexactness of the approximations in satisfying eqns (A14), (1) and (2).

Closer approximations are obtained by using

\[
(M_i)_{N+1} = (M_i)_N - \Delta M_i
\]

(A18)

\[
(M_e)_{N+1} = (M_e)_N - \Delta M_e
\]

(A19)

and

\[
(G)_{N+1} = (G)_N - \Delta G
\]

(A20)

where suffixes \( N \) and \( N + 1 \) refer to successive approximations, and \( \Delta M_i, \Delta M_e \) and \( \Delta G \) are given by three linear equations conveniently written in the form

\[
\begin{bmatrix}
\frac{\partial g_1}{\partial M_i} & \frac{\partial g_1}{\partial M_e} & \frac{\partial g_1}{\partial G} \\
\frac{\partial g_2}{\partial M_i} & \frac{\partial g_2}{\partial M_e} & \frac{\partial g_2}{\partial G} \\
\frac{\partial g_3}{\partial M_i} & \frac{\partial g_3}{\partial M_e} & \frac{\partial g_3}{\partial G}
\end{bmatrix}
\begin{bmatrix}
\Delta M_i \\
\Delta M_e \\
\Delta G
\end{bmatrix}
= \begin{bmatrix}
g_1 \\
g_2 \\
g_3
\end{bmatrix}
\]

(A21)
The derivatives are
\[
\begin{align*}
\mathcal{D}_1/\partial M_i &= \left( W/2kL \right) \left[ (k + 1)/M_i - 2/M_i^{-3} - (k^2 - 1)M_i/(2 + (k - 1)M_i^2) \right] \\
\mathcal{D}_1/\partial M_e &= -\left( W/2kL \right) \left[ (k + 1)/M_e - 2/M_e^{-3} - (k^2 - 1)M_e/(2 + (k - 1)M_e^2) \right] \\
\mathcal{D}_G &= 0 \\
\mathcal{D}_2/\partial M_i &= \left[ 1 - (k + 1)M_i^2/(2 + (k - 1)M_i^2) \right] \left[ 1 + M_i^2(k - 1)/2 \right]^{-1}\left(k+1\right)/2^{k-1} \\
\mathcal{D}_2/\partial M_e &= 0 \\
\mathcal{D}_3/\partial M_i &= 0 \\
\mathcal{D}_3/\partial M_e &= \left[ 1 - (k + 1)M_e^2/(2 + (k - 1)M_e^2) \right] \left[ 1 + M_e^2(k - 1)/2 \right]^{-1}\left(k+1\right)/2^{k-1} \\
\mathcal{D}_G &= -\left( p_{\infty} \right) \left( RT_e/k \right)^{1/2}
\end{align*}
\] (A22)

The iterative solutions for \( M_i, M_e \) and \( G \) are continued until the corrections \( \Delta M_i, \Delta M_e \) and \( \Delta G \) are sufficiently small.