Fundamental Characteristics and Behavior of Reinforced Concrete Bridge Piers Subjected to Reversed Cyclic Loading

S. H. Rizkalla, F. Saadat, and T. Higai

ABSTRACT

Sixteen large-scale reinforced concrete specimens without web reinforcements were tested to study the behavior of typical bridge piers subjected to deflection reversals that were large enough to cause extensive yielding of the longitudinal reinforcement. The different parameters considered included shear span-to-depth ratio, percentage of longitudinal reinforcement, frequency of the applied load, and axial compressive stresses. Each specimen was subjected to a multiple of three cycles of deflection reversals, in increments equal to the yield deflection until failure. Based on parametric studies of the experimental results, a nondimensional factor was introduced to describe the fundamental behavior of such members. The proposed characteristic factor was used to evaluate the ductility, maximum shear stress, energy absorption-dissipation capacities, and the equivalent viscous damping coefficient. The three modes of failure observed were classified according to the range of the proposed factor, and also to the maximum intensity of shear stresses. Based on the experimental results, an expression was introduced to predict the equivalent flexural stiffness in the post-yielding range.

Bridge piers are commonly lightly reinforced in both longitudinal and transverse directions. Unlike beams and columns, they fall into different categories in terms of shear span-to-depth ratio, as related to the percentage of reinforcement and level of axial compressive stress. The main objective of this paper is to study the behavior of typical bridge piers when they are subjected to reversed cyclic deflections large enough to cause extensive yielding of the longitudinal reinforcement. Sixteen large-scale, reinforced concrete specimens without web reinforcements were tested. The specimens were representative of typical bridge piers in terms of material properties, section properties, and construction details. The different parameters considered in this program are the shear span-to-depth ratio (a/d), percentage of longitudinal reinforcement (p), frequency of the applied load, and level of axial compressive stresses (σ).

The experimental program was divided into three major series, as given in Table 1. The objective for subseries I-A was to examine the effect of shear span-to-depth ratio, a/d, on the mode of failure and ductility. For these specimens, the percentage of longitudinal reinforcement was varied to maintain a constant ratio between the shear strength under monotonic load, v_0, and the shear stress at yielding of the longitudinal reinforcement, v_y. In subseries I-B, seven specimens were tested to examine the effect of the percentage of steel, p. The effect of the load frequency was examined in Series II. Finally, two specimens were tested in Series III with axial compressive force to simulate the actual conditions of the prototype.

TEST SPECIMENS

The relationship between a tested specimen and a typical bridge pier is shown in Figure 1. The dimensions and reinforcement details of a typical specimen are also shown in the same figure. The effective depth, d, was varied to allow for the variation of shear span-to-depth ratio, a/d, between 3.29 and 6.05. The percentage of steel, p, was also selected to cover the typical range of such members, as given in Table 1. The end block was designed to simulate the boundary conditions provided by the foundation of typical reinforced concrete bridge piers. All concrete used in fabrication of the specimens was designed for a nominal ultimate strength of 30 MPa and was ready-mixed from a local concrete mix plant. The reinforcement used consisted of hot, deformed bar grade of 300 MPa.

For the two specimens in Series III, the axial compression load was applied using an especially designed load frame equipped with two hydraulic jacks and pin connection to allow for the rotation of the cantilever in the vertical plane, as shown in Figure 2. A bracing system provided by Teflon strips was attached at the end of the cantilever to minimize its lateral rotation.

TESTING APPARATUS AND PROCEDURE

The vertical reversing loads and deflections were applied using an electric-servo, closed-loop, MTS testing system. The specimen was loaded downward (negative load) up to a load equal to the calculated yield load, P_y. The corresponding deflection at the location of the load, δ_y, was measured. The specimen was unloaded and then loaded upward (positive load) to a deflection equal to the yield deflection. The specimen was unloaded and the same procedure was repeated for two more cycles of deflection reversals, as shown in Figure 3. After three cycles, the deflection was increased by increments of δ_y until the load-resisting capacity of
### TABLE 1 Variables Considered in the Experimental Program

<table>
<thead>
<tr>
<th>Series</th>
<th>Specimen No.</th>
<th>b x d (mm)</th>
<th>Longitudinal Reinforcement</th>
<th>p (percent)</th>
<th>s/d</th>
<th>$P_y$ (kN)</th>
<th>$v_p$ (MPa)</th>
<th>$v_H$ (MPa)</th>
<th>$v_p/v_H$</th>
<th>Frequency of Loading (Hz)</th>
<th>Axial Compression Stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-A</td>
<td>1-1</td>
<td>500x350</td>
<td>3-20 M</td>
<td>0.51</td>
<td>3.29</td>
<td>83.9</td>
<td>0.80</td>
<td>0.48</td>
<td>1.67</td>
<td>0.0004</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>1-2</td>
<td>500x350</td>
<td>3-20 M</td>
<td>0.96</td>
<td>5.0</td>
<td>65.8</td>
<td>0.96</td>
<td>0.57</td>
<td>1.68</td>
<td>0.0004</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>1-3</td>
<td>500x190</td>
<td>4-20 M</td>
<td>1.26</td>
<td>6.05</td>
<td>57.8</td>
<td>1.06</td>
<td>0.61</td>
<td>1.74</td>
<td>0.0004</td>
<td>None</td>
</tr>
<tr>
<td>I-B</td>
<td>1-4</td>
<td>500x190</td>
<td>6-20 M</td>
<td>1.89</td>
<td>6.05</td>
<td>83.7</td>
<td>1.27</td>
<td>0.88</td>
<td>1.44</td>
<td>0.0004</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>1-5</td>
<td>500x190</td>
<td>3-20 M</td>
<td>0.95</td>
<td>6.05</td>
<td>43.8</td>
<td>1.02</td>
<td>0.46</td>
<td>2.22</td>
<td>0.0004</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>1-6</td>
<td>500x190</td>
<td>5-20 M</td>
<td>1.58</td>
<td>6.05</td>
<td>70.8</td>
<td>1.12</td>
<td>0.75</td>
<td>1.49</td>
<td>0.0004</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>1-7</td>
<td>500x350</td>
<td>2-20 M</td>
<td>0.40</td>
<td>3.29</td>
<td>66.1</td>
<td>0.74</td>
<td>0.38</td>
<td>1.95</td>
<td>0.0004</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>1-8</td>
<td>500x350</td>
<td>1-10 M</td>
<td>0.46</td>
<td>3.29</td>
<td>74.8</td>
<td>0.77</td>
<td>0.43</td>
<td>1.79</td>
<td>0.0004</td>
<td>None</td>
</tr>
<tr>
<td>II</td>
<td>1-9</td>
<td>500x350</td>
<td>2-20 M</td>
<td>0.86</td>
<td>3.29</td>
<td>140.7</td>
<td>0.99</td>
<td>0.80</td>
<td>1.24</td>
<td>0.0004</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>1-10</td>
<td>500x350</td>
<td>3-20 M</td>
<td>0.64</td>
<td>4.11</td>
<td>68.0</td>
<td>0.89</td>
<td>0.49</td>
<td>1.82</td>
<td>0.0004</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>2-1</td>
<td>500x350</td>
<td>2-20 M</td>
<td>0.51</td>
<td>3.29</td>
<td>83.8</td>
<td>0.77</td>
<td>0.48</td>
<td>1.60</td>
<td>0.01</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>2-2</td>
<td>500x350</td>
<td>3-20 M</td>
<td>0.51</td>
<td>3.29</td>
<td>83.8</td>
<td>0.77</td>
<td>0.48</td>
<td>1.60</td>
<td>0.05</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>2-3</td>
<td>500x350</td>
<td>3-20 M</td>
<td>0.51</td>
<td>3.29</td>
<td>83.8</td>
<td>0.78</td>
<td>0.48</td>
<td>1.60</td>
<td>0.1</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>3-1</td>
<td>500x350</td>
<td>3-20 M</td>
<td>0.51</td>
<td>3.29</td>
<td>108.7</td>
<td>0.84</td>
<td>0.62</td>
<td>1.35</td>
<td>0.0004</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>3-3</td>
<td>500x190</td>
<td>4-20 M</td>
<td>1.26</td>
<td>6.05</td>
<td>58.8</td>
<td>1.10</td>
<td>0.62</td>
<td>1.77</td>
<td>0.0004</td>
<td>None</td>
</tr>
</tbody>
</table>

![Typical Bridge](image)

**FIGURE 1** Relationship between a typical specimen and bridge piers.

The specimen was less than the yield load, indicating failure of the specimen.

For Series II, the preceding procedures were used for the first half-cycle followed by a fully reversed sine wave loading under the deflection control mode. For Series III, the axial load was applied and maintained constant by using a pressure regulator before the application of the yield load and the measurement of the yield deflection.

The strains of the longitudinal steel were measured using electric resistance strain gauges. The average strain was also measured using a linear variable differential transformer (LVDT), which was mounted on steel studs projecting from the longitudinal reinforcement, as shown in Figure 4. By using the same technique, the average diagonal and vertical strains were measured to determine the average shear strain. Loads and deflections at the end of the cantilever part of the specimen were recorded continuously by an X-Y recorder. Crack propagation was observed using a magnifying lens, and sketches were plotted as the cracks progressed.

**TEST RESULTS**

Material properties and the experimental results for the sixteen specimens are given in Table 2. Because of the nature of the applied reversed loading, the diagonal shear cracks intersected each other to form an X-shaped crack for most of the specimens. The load deflection hysteresis response for a typical specimen, 1-4, is shown in Figure 5.

**Ductility**

The ductility factor, $u$, defined as the ratio between the maximum deflection before failure, $\delta_f$, and the deflection at yielding, $\delta_y$, is given for all the specimens in Table 2. In this table, specimen 1-3, which failed to sustain a load higher than the yield load ($P_y$) during the first cycle of the fourth deflection increment at a deflection of 3.74 $\delta_y$, has a ductility factor equal to 3. However, specimen 1-4, which failed during the same cycle ($46\delta_y$), attains a
ductility factor of 2 because the load carrying capacity for this specimen dropped below $P_y$ at a deflection of $3\alpha_y$. In general, the results indicated that ductility decreases with the increase of the percentage of longitudinal steel and the yield strength of the steel, $f_y$. It is also clear that the ductility is influenced by the shear span-to-depth ratio, $a/d$, and the tensile strength of the concrete, $f_t$, as all the specimens failed in shear (1).

The relationship between the ductility factor, $\nu$, and the previously mentioned parameters was examined using the experimental data for all the specimens tested in Series I of this program. Based on a multiple linear regression analysis, the following relationship was obtained:

$$\nu = 0.38 (p_{fy})^{-1.74} (a/d)^{1.92} (f_c) 1.43$$  \hspace{1cm} (1)

The mean and the standard deviation values were found to be 1.00 and 0.11, respectively. The tensile strength, $f_t$, could be evaluated based on the measured concrete splitting strength, $f_{sp}$, or the ultimate compressive strength, $f_c$, as follows (2):

$$f_t = 0.6 f_{sp}$$  \hspace{1cm} (2a)

or

$$f_t = f_c^{1/3}$$  \hspace{1cm} (2b)

The powers of the different variables included in the parametric relationship, Equation 1, suggest the possibility of using a nondimensional characteristic factor, $K$, to predict the ductility, where

$$K = [(a/d) f_c / p_{fy}]$$  \hspace{1cm} (3)

The relationship between the ductility factor, $\nu$, and the proposed characteristic factor, $K$, for all the specimens in Series I of this program is shown in Figure 6. Based on a best-fitting curve for the experimental data, the following relationship is proposed:

$$\nu = 1/(0.8 - 0.13 K)$$  \hspace{1cm} (4)

**TABLE 2 Test Results**

<table>
<thead>
<tr>
<th>Series No.</th>
<th>Specimen</th>
<th>$f_c$ (MPa)</th>
<th>$f_y$ (MPa)</th>
<th>$\delta$ (mm)</th>
<th>$P_{max}$ (kN)</th>
<th>$\nu_p$ (MPa)</th>
<th>$\nu_{u/py}$</th>
<th>Failure Cycle</th>
<th>Yielding Location</th>
<th>$\mu$</th>
<th>NED</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-A</td>
<td>1-1</td>
<td>37</td>
<td>343</td>
<td>2.24</td>
<td>4.33</td>
<td>3.77</td>
<td>-103.76</td>
<td>0.59</td>
<td>1.23</td>
<td>-63y/1</td>
<td>1d</td>
</tr>
<tr>
<td>1-2</td>
<td>35</td>
<td>341</td>
<td>2.19</td>
<td>3.19</td>
<td>6.70</td>
<td>-73.43</td>
<td>0.64</td>
<td>1.12</td>
<td>-55y/1</td>
<td>1.5d</td>
<td>3</td>
</tr>
<tr>
<td>1-3</td>
<td>36</td>
<td>343</td>
<td>2.28</td>
<td>3.17</td>
<td>8.56</td>
<td>68.29</td>
<td>0.72</td>
<td>1.18</td>
<td>+45y/1</td>
<td>1d</td>
<td>3</td>
</tr>
<tr>
<td>1-4</td>
<td>40</td>
<td>341</td>
<td>2.58</td>
<td>2.4</td>
<td>8.40</td>
<td>80.64</td>
<td>0.93</td>
<td>1.06</td>
<td>+45y/1</td>
<td>2d</td>
<td>2</td>
</tr>
<tr>
<td>1-5</td>
<td>42</td>
<td>341</td>
<td>2.40</td>
<td>4.54</td>
<td>6.90</td>
<td>-82.52</td>
<td>0.55</td>
<td>1.00</td>
<td>-65y/2</td>
<td>2d</td>
<td>5</td>
</tr>
<tr>
<td>1-6</td>
<td>33</td>
<td>342</td>
<td>2.04</td>
<td>2.28</td>
<td>8.50</td>
<td>75.34</td>
<td>0.80</td>
<td>1.07</td>
<td>+45y/1</td>
<td>2d</td>
<td>2</td>
</tr>
<tr>
<td>1-7</td>
<td>38</td>
<td>342</td>
<td>2.16</td>
<td>5.14</td>
<td>2.66</td>
<td>-79.15</td>
<td>0.45</td>
<td>1.18</td>
<td>-65y/2</td>
<td>2d</td>
<td>5</td>
</tr>
<tr>
<td>1-8</td>
<td>38</td>
<td>343</td>
<td>2.52</td>
<td>5.24</td>
<td>2.77</td>
<td>-92.15</td>
<td>0.33</td>
<td>1.23</td>
<td>-65y/2</td>
<td>2d</td>
<td>5</td>
</tr>
<tr>
<td>1-9</td>
<td>41</td>
<td>343</td>
<td>2.52</td>
<td>2.82</td>
<td>4.68</td>
<td>145.55</td>
<td>0.83</td>
<td>1.04</td>
<td>+45y/1</td>
<td>1d</td>
<td>2</td>
</tr>
<tr>
<td>1-10</td>
<td>41</td>
<td>343</td>
<td>2.64</td>
<td>4.88</td>
<td>4.21</td>
<td>-78.83</td>
<td>0.37</td>
<td>1.16</td>
<td>-65y/1</td>
<td>1d</td>
<td>6</td>
</tr>
<tr>
<td>1-11</td>
<td>40</td>
<td>343</td>
<td>2.00</td>
<td>3.76</td>
<td>3.74</td>
<td>-89.5</td>
<td>0.59</td>
<td>1.23</td>
<td>-65y/1</td>
<td>1d</td>
<td>3</td>
</tr>
<tr>
<td>1-12</td>
<td>34</td>
<td>343</td>
<td>2.00</td>
<td>3.76</td>
<td>3.80</td>
<td>-107.5</td>
<td>0.61</td>
<td>1.27</td>
<td>-65y/2</td>
<td>1d</td>
<td>4</td>
</tr>
<tr>
<td>1-13</td>
<td>34</td>
<td>343</td>
<td>2.04</td>
<td>3.86</td>
<td>3.33</td>
<td>-101.5</td>
<td>0.58</td>
<td>1.21</td>
<td>-65y/1</td>
<td>1d</td>
<td>4</td>
</tr>
<tr>
<td>1-14</td>
<td>35</td>
<td>350</td>
<td>2.23</td>
<td>3.06</td>
<td>6.54</td>
<td>-74.0</td>
<td>0.78</td>
<td>1.26</td>
<td>-65y/2</td>
<td>2d</td>
<td>5</td>
</tr>
<tr>
<td>1-15</td>
<td>36</td>
<td>343</td>
<td>2.04</td>
<td>3.85</td>
<td>3.55</td>
<td>+122.12</td>
<td>0.70</td>
<td>1.13</td>
<td>-65y/2</td>
<td>2d</td>
<td>4</td>
</tr>
<tr>
<td>1-16</td>
<td>36</td>
<td>343</td>
<td>2.52</td>
<td>3.54</td>
<td>8.14</td>
<td>-73.62</td>
<td>0.77</td>
<td>1.13</td>
<td>-55y/2</td>
<td>2d</td>
<td>4</td>
</tr>
</tbody>
</table>
FIGURE 5 Typical load-deflection hysteresis.

FIGURE 6 Relationship between ductility and the characteristic factor, $K$.

The preceding relationship is shown also in Figure 6 as a solid line. The mean and the coefficient of variance values were found to be 1.00 and 9 percent, respectively, clearly indicating a high degree of predictive accuracy of the proposed expression. Specimens in Series II are also shown in the same figure.

The proposed relation suggests a minimum value of 2.3 for the characteristic factor, $K$, which corresponds to an integer value for the ductility factor of 2. The proposed expression also provides the maximum value for $K$ of 5.3, which corresponds to a ductility factor of 9 for possible shear failure.

MODES OF FAILURE

The appearance of the crack configuration failed to indicate a clear trend that could be used to classify the mode of failure for such members. The location of yielding of the longitudinal reinforcement was the only common observation that could be used to classify the different mechanisms. By using the proposed characteristic factor, $K$, it was possible to classify the modes of failure accordingly, as shown in Figure 6. Based on the range of the $K$-values, the critical crack pattern and the extent of yielding of the longitudinal reinforcement could be predicted. The proposed classification of mode of failure can be summarized as follows:

Mode 1

Failure is due mainly to a large vertical relative movement at the maximum moment section, caused by the localization of the yielding of the longitudinal reinforcement at this section, as shown in Figure 7 for specimen 1-7. This mode is typical for $K$-values between 4.8 and 5.3.

FIGURE 7 Final crack pattern for Specimen 1-7, 95y/3.

Mode 2

This failure mode is characterized by yielding of the longitudinal reinforcement within a distance $d$ from the maximum moment section. The failure is due mainly to widening of the x-shaped crack or buckling of the longitudinal reinforcement within the same distance, as shown in Figures 8 and 9 for specimens 1-1 and 1-5, respectively. This mode is typical for $K$-values between 4 and 4.8.
EFFECT OF AXIAL LOAD

To study the effect of axial load, specimen 3-1 and 3-3, used as representatives of the lower and higher ranges of a/d, were tested in this program. The two specimens were subjected to an axial compressive stress of 0.98 MPa, which is typical for short and intermediate bridge piers (3). The results indicated that the presence of low-intensity, axial compressive stresses appeared to enhance the maximum shear resistance and ductility of such members (4). This observation is contrary to the fact that high-intensity axial stresses decrease the ductility. It was also observed that axial compressive stresses affected the modes of failure. However, no general conclusion can be made at this stage because of the limited ranges considered in this program.

ENERGY ABSORPTION CAPACITY

In general, the magnitude of the energy absorption capacity, \( W_a \), provides a means to evaluate the response of a structure to the various loading histories. By definition, this energy could be determined based on the actual area under the load-deflection hysteresis preceding failure deflection increment. The relationship between the normalized energy absorption capacity with respect to the product of yield load and deflection, \( I_w \), and the proposed characteristic factor, \( K \), for all the specimens in Series I could be mathematically expressed as follows:

\[
I_w^* = 1.82 \exp(0.86K) \tag{5}
\]

The mean and coefficient of variance values for the preceding relationship are 1.02 and 14 percent, respectively.

The work index \( I_w \), introduced by Gossain et al. (5), can be expressed as

\[
I_w = \frac{\sum (P_i \delta_i)}{P_y \delta_y} \tag{6}
\]

where \( P_i \) and \( \delta_i \) are the maximum applied load and deflection at the \( i \)th cycle, and \( n \) represents the number of cycles preceding the failure deflection increment. Based on the experimental results, the relationship between \( I_w \) and \( I_w \) was examined and found
to be linear. The obtained mean and coefficient of variance values of 1.04 and 7 percent, respectively, indicate an excellent agreement between the two indices. This would justify the use of the simplified work index, $I_w$, in lieu of $I_y$, to represent the actual absorbed energy, $W_y$. The relationship between $I_w$ and the proposed nondimensional factor $K$ for all the specimens in Series I is shown in Figure 11. The solid line represents the best-fitting curve to the experimental data, and can be mathematically expressed as

$$I_w = K/(0.41 - 0.07 K)$$  \hspace{1cm} (7)

The mean and coefficient of variance of 1.01 and 9 percent, respectively, were obtained.

To minimize the effect of the loading pattern used in this program, an average work index, $I_{wave}$, could be used as follows (8):

$$I_{wave} = I_w/n$$  \hspace{1cm} (8)

The relationship between the ductility factor and the average work index for all the specimens tested in Series I without web reinforcement is shown in Figure 12. The solid line represents the best-fitting curve to the experimental data, which can be mathematically expressed as

$$\mu = 1.72 I_{wave} - 0.72$$  \hspace{1cm} (9)

The mean and coefficient of variance of 1.00 and 5 percent, respectively, were obtained. The comparison between the preceding equation for piers without web reinforcement and the expression proposed by Arakawa (5) for columns with 0.3-percent transverse reinforcement is also shown in Figure 12. The proposed relationship complements Arakawa's findings by introducing the lower bound for the family of curves with different percentages of web reinforcement.

DEFINITION OF DUCTILITY

In general, the magnitude of the energy dissipation capacity provides a measure for the inelastic performance of a structure under reversed loads. By definition, this energy could be evaluated based on the areas enclosed by the load-deformation hysteresis loops preceding failure deflection. The relationship between the normalized energy dissipation, NED, with respect to the linearized applied energy of the first cycle, $F_y x (dy/2)$, and the ductility factor, $\nu$, for all the specimens in Series I, is shown in Figure 13.

$$NED = 1.8 (\mu)^{2.19}$$  \hspace{1cm} (10)

The mean and the coefficient of variance are found to be 1.01 and 9 percent, respectively. Thus, for a given ductility, which can be predicted by using Equation 3, the normalized energy dissipation can be determined.

MAXIMUM SHEAR STRESS

Based on the measured data, the maximum shear stress, $\sigma_s$, is normalized with respect to the concrete tensile strength $f_y$, and related to the characteristic factor, $K$, for all the specimens in Series I, (Figure 14). The solid line represents the best-
fitting curve for the experimental data, which can be expressed mathematically as

\[ v_0 / f_t = 0.71/k^{0.74} \]  

(11)

The high predictive accuracy of this proposed expression is reflected by the values for the mean and the coefficient of variance of 0.99 and 2.5 percent, respectively. Specimens in Series II are also shown in the same figure. Based on this relationship, it is recommended that for bridge piers without web reinforcement, the intensity of shear stresses corresponding to the yield load be limited to 45 percent of the shear strength of the concrete under monotonic load, \( v_0 \). This would ensure reasonable performance under large deflection reversals (\(|\gamma|\)). The modes of failure were also classified according to the intensity of maximum shear stresses. By using the same specified ranges of K-value for each failure mode, the ranges of the intensity of maximum shear stresses are established for the three proposed modes (Figure 14).

The maximum shear stress resistance, \( v_{\text{res}} \), of a structural member depends also on the degree of severity of the applied energy represented by the work index, \( I_w \). The relationship between the normalized maximum shear stress with respect to the tensile strength of concrete, \( v_{\text{res}} / f_t \), and the work index based on the experimental results of Series I is shown in Figure 15. The solid line representing the best-fitting curve to the experimental data can be expressed mathematically as

\[ v_{\text{res}} / f_t = 0.6 / 10^{0.23w} \]  

(12)

The obtained mean and coefficient of variance values are 1.01 and 6 percent, respectively. By using this relationship, it is possible to evaluate the shear stress of a pier without web reinforcement, \( v_{\text{res}} \), for any level of desired deformation capacity. For example, assume that a pier is required to provide energy absorption such that it can withstand a deflection of five times the yield deflection with a deflection history as shown in Figure 3, and that the load capacity should always be more than the yield load. The corresponding work index in this case is approximately equal to 45, which would translate into a maximum shear stress of approximately 25 percent of the tensile strength of the concrete according to Equation 12. This value could also represent the concrete contribution in resisting shear for pier structural members with web reinforcement.

**EQUIVALENT VISCOS DAMPING COEFFICIENT**

The use of the equivalent viscous damping coefficient, \( \zeta_{eq} \), has the advantage of linearizing the equation of motion. This coefficient may approximate the other forms of damping if the dissipative energy is small. The coefficient, \( \zeta_{eq} \), can be expressed in terms of the energy dissipation capacity, \( W_{d} \), and the linear strain energy stored at maximum displacement, \( W_{s} \), as follows (7):

\[ \zeta_{eq} = W_{d} / 4 \pi W_{s} \]  

(13)

The equivalent viscous damping at the ultimate stage, as related to the characteristic factor (K) for all the specimens tested in Series I, is shown in Figure 17. It is clear that the histories of all these specimens can be represented by a single curve independent of their fundamental variables, as shown by the solid line. By using linear regression analysis, the relationship can be expressed mathematically as

\[ \zeta_{eq} = 0.12 (K^{1/3}) \]  

(14)

The accumulated equivalent viscous damping coefficient at different deflection increments for all the specimens tested in Series I is shown in Figure 18. It is clear that the histories of all these specimens can be represented by a single curve independent of their fundamental variables, as shown by the solid line. By using linear regression analysis, the relationship can be expressed mathematically as

\[ \zeta_{eq} = 0.22 - (0.15 / \gamma) \]  

(15)

in which \( \gamma \) is the deflection limit at which the accumulated equivalent viscous damping coefficient is sought. In the same figure, the dotted line represents the relationship proposed by Gulkan et al. (8) for reinforced concrete columns with web reinforcement under high-frequency loading conditions.
For deflection calculation at service load conditions, the flexural stiffness, $E_1$, could be predicted by using the effective moment of inertia, $I_{eq}$, and the concrete modulus of elasticity, $E_c$, as proposed by the ACI code (9). The comparison between the computed and measured values proved to be in good agreement up to 70 percent of the yield load (10). However, at higher load levels, the scatter appears and increases as the load approaches the ultimate load under monotonic loading condition.

Under deflection reversals large enough to cause extensive yielding of the longitudinal reinforcement, extensive cracking of the concrete occurs both at the top and the bottom surfaces of the beam, leading to a substantial reduction in the stiffness. The stiffness changes continuously with the variation of the applied load. However, the slope of the secant line at any point on the load-deflection curve could be used to represent the equivalent flexural stiffness, $E_1$, at that level. The stiffness ratios, $\phi$, between the equivalent flexural stiffness, $E_1$, and the effective flexural stiffness, $E_{c1}E_{eff}$, were calculated for all the specimens tested in Series I in this program. Based on the multiple regression analysis of the data, the relationship between the stiffness ratio, $\phi$, $\delta/\delta_y$, and $a/d$ were established as shown in Figure 18 by the solid line, which can be expressed mathematically in the following form (4):

$$\phi = 0.17 (a/d)^{0.78} (\delta/\delta_y)^{0.85} \quad (16)$$

The mean and coefficient of variance of 0.99 and 12.9 percent, respectively, were obtained. By using the preceding equation, the equivalent flexural stiffness, $E_1$, for a given, reinforced concrete bridge pier at any desired deflection level, $\delta$, in the post-yielding range could be evaluated.

**SUMMARY AND CONCLUSIONS**

The following observations and conclusions are based on the experimental program discussed in this study:

1. The fundamental behavior of bridge piers can be fully described using a proposed nondimensional characteristic factor, $K$, as follows:

$$K = \frac{(a/d) f_y/f_{cy}}{} \quad (17)$$

2. The proposed factor, $K$, was used to derive expressions that could be used to evaluate the ductility, maximum shear stress, energy dissipation and absorption capacities, and the ultimate, equivalent, viscous damping coefficient of such members.

3. By using the proposed characteristic factor, $K$, it was possible to classify the failure mechanisms into three different distinct modes of failure. This classification was accomplished parallel to a rational categorization of these failure modes based on the maximum intensity of the applied shear stresses.

4. The load frequency within the limited range of this program is considered to have an insignificant effect on the mode of failure, maximum shear stress, and ductility.

5. The results of the two specimens tested in the presence of the axial compressive load indicated that the low intensity of axial compressive stresses enhance the shear strength and ductility.

6. For the given loading history, a relationship between the ductility factor and the average work index $W_{ave}$ was developed to allow for the prediction of the deformation capacity of such members subjected to various degrees of severity of the applied load.
FIGURE 18  Relationship between stiffness ratio, and displacement ductility ratio, $S/y$.

7. Expressions were proposed for the prediction of the equivalent viscous damping coefficient and the equivalent flexural stiffness of such members in the post-yielding range.

ACKNOWLEDGMENTS

This study was carried out by the Department of Civil Engineering at the University of Manitoba with financial assistance from the Natural Sciences and Engineering Research Council of Canada.

REFERENCES