How to Prepare Mathematics Homework

Homework assignments in mathematics serve a number of important functions:

1. They give the students practice with the technical details of the material they have learned in class, and help the students learn this material better.

2. They can be used as study tools for midterms, finals, and qualifying exams.

3. They give students practice at communicating mathematics. As a professional mathematician (either in academia or industry) one of the main roles of your job will be to explain mathematics to others.

4. They allow the professor to assess student performance.

Points 1,2, and 3 above are certainly the most important reasons for assigning homework. With these ideas in mind, I will describe criteria for preparing a well-written mathematics homework.

Format of homework.

1. Make sure all solutions are turned in in order, pages are stapled, with sufficient margins so I can write comments.

2. Write out the statement of each problem before writing the solution.

3. Problem solutions should be written in paragraph format, not just a sequence of equations.

4. Please indicate on each problem who you worked with on the problem, or if you found the solution to the problem in a book or online.

Style of homework.

1. You should imagine that you are writing your homework solutions for another student in the class. Your solution should be able to convince another student that your solution is correct, without any oral explanation from you. Another student should be able to understand your solution completely via the written word, and whatever diagrams or tables you create in your solution.

2. The level of detail that I write on the board in class is usually not enough for a complete homework solution. Lectures use much shorthand that is inappropriate for a homework solution. Lectures often contain oral justifications of statements, which would need to be included as carefully written sentences in a homework.

3. Proof is the name of the game in advanced mathematics homework problems in pure mathematics. This includes problems that ask you a question of classification (e.g. "Determine all things that satisfy property X"). You must not only provide the classification, but prove that all things in your list satisfy property X and prove that there are no other things that satisfy property X.

4. Some proofs might involve a computation, possibly using a computer.

5. It is a matter of taste how much detail is enough in your proofs. It is generally safer to err on the side of too much detail rather than too little.

Grading of Homework

Each homework assignment is worth twelve points. Six of the twelve points will be determined by whether or not the assignment is complete and well-prepared, in particular, satisfying the formatting and style requirements above. I will choose two problems on each assignment to grade carefully for correctness. Each of these solutions will be worth three points.

Specific Comments for Math 521/721 Homework

In this course, we will build the foundations of algebra "from the ground up". I assume throughout that you are familiar with the following elementary notions: matrices, vector spaces, linear transformations, functions, the notion of a field (like the rational numbers $\mathbb{Q}$, the real numbers $\mathbb{R}$, and the complex numbers $\mathbb{C}$), the integers $\mathbb{Z}$, the division algorithm for integers. You should not use other concepts from abstract algebra until we introduce and derive them in class, or the concept is introduced and defined in the corresponding section of the book. A typical example is that many people try use Lagrange's theorem before we have gone over it in class.

The purpose of this is to force you to practice working with the elementary definitions. This practice is extremely valuable for when you will teach courses in algebra in your future career as a mathematics professor.

Example Homework Problem Solution

Sec 2.2 Problem 12: Describe all groups which contain no proper subgroup.

I discussed this problem with John Q. Public and Jane Doe.

Solution. The only groups which contain no proper subgroup are the trivial group $\{1\}$ and the cyclic groups C_p of prime order. The trivial group has no proper subgroup because its only subgroup is $\{1\}$. To see that the cyclic group C_p has no proper subgroup, suppose by way of contradiction that $H \subseteq C_p$ is a proper subgroup. Since H is proper, it contains some x^i with $i \in \{1, 2, \ldots, p-1\}$. The smallest $k > 0$ such that $(x^i)^k = 1$ is $k = p$, since $x^{ik} = 1$ if and only if $p|ik$, and i is relatively prime to p. Hence $\langle x^i \rangle = C_p$ so $H = C_p$. This contradicts the fact that H was a proper subgroup.

We must show that these are the only possibilities. Consider any other group G which has no proper subgroups. Suppose that G is not the trivial group, and let $g \in G$ with $g \neq 1$. Since G has no proper subgroups, we must have $\langle g \rangle = G$. This shows that G is a cyclic group. The group G could not be an infinite cyclic group, because in that case $\langle g^2 \rangle \subset \langle g \rangle$ is a proper subgroup. Thus, G must be isomorphic to a finite cyclic group C_r. However, if r is composite ($r = st$), then $\{1, g^t, g^{2t}, \cdots, g^{(s-1)t}\}$ is a proper subgroup. This shows that G must be either the trivial group or cyclic of prime order. $\square$