**Compound Interest Problems:**

1. A man invests $\$10,000$ in an account that pays $8.5\%$ interest per year, compounded quarterly. What is the amount of money that he will have after 3 years?

   Information given in problem:
   $P = \$10,000$, $r = 0.085$, $t = 3$, and $n = 4$.
   Using the formula
   $$A = P\left(1 + \frac{r}{n}\right)^{nt},$$
   we have
   $$A = 10,000\left(1 + \frac{0,085}{4}\right)^{(4)(3)},$$
   so the man will have $\$12,870.19$ after 3 years

2. A sum of $\$5000$ is invested at an interest rate of $9\%$ per year. Find the time required for the money to double if the interest is compounded:

   (a) Semi-annually

   We have $P = \$5000$, $r = 0.09$, and $n = 2$. We want the principal to double, so $A = 2P = \$10,000$. Then, using
   $$A = P\left(1 + \frac{r}{n}\right)^{nt},$$

we have that

$$
\begin{aligned}
10000 &= 5000\left(1+\frac{0.09}{n}\right)^{2t}\\
2 &= \left(1+\frac{0.09}{n}\right)^{2t}\\
ln(2) &= ln\left(1+\frac{0.09}{n}\right)^{2t}\\
ln(2) &= 2tln\left(1+\frac{0.09}{n}\right)^{2t}\\
t &= \frac{ln(2)}{2ln\left(1+\frac{0.09}{n}\right)^{2t}} \approx 7.9.
\end{aligned}
$$

This tells us that it will take approximately 7.9 years for the investment to double when interest is compounded semi-annually.

(b) Continuously

$A$, $P$, and $r$ are the same as in part (a). With continuous compounding, we always use $A = Pe^{rt}$, so in this situation we have

$$
\begin{aligned}
10000 &= 5000e^{0.09t}\\
2 &= 5000e^{0.09t}\\
ln(2) &= .09t\\
t &= \frac{ln(2)}{.09} \approx 7.7.
\end{aligned}
$$

This means that it takes about 7.7 years for the investment to double when interest is compounded continuously.

3. Find the effective rate of interest for an investment that earns $5\frac{1}{2}\%$ per year, compounded continuously.

We are not given a value of $P$ in this problem, so either pick a value for $P$ and stick with that throughout the problem, or just let $P = P$. We have that $t = 1$, and $r = .055$. To find the effective rate of interest, first find out how much money we have after one year:

$$A = Pe^{rt}$$
$$A = Pe^{(.055)(1)}$$
$$A = 1.056541P.$$

Therefore, after 1 year, whatever the principal was, we now have $1.056541P$. Next, find out how much interest was earned, $I$, by subtracting the initial amount of money from the final amount:

$$I = A - P$$
$$= 1.056541P - P$$
$$= .056541P.$$

Finally, to find the effective rate of interest, use the simple interest formula, $I = Prt$. So,

$$I = Pr(1) = .056541P$$
$$.056541 = r.$$

Therefore, the effective rate of interest is 5.65%.

4. How much money should I save in an account paying 5% interest compounded monthly if I want to have \$6000 in 6 months?

Given: $r = .05$, $n = 12$, $t = .5$ (remember $t$ must be in YEARS), and $A = 6000$. Find $P$ using the compound interest formula

$$\begin{aligned} A &= P\left(1+\frac{r}{n}\right)^{nt} \\ 6000 &= P\left(1+\frac{.05}{12}\right)^{(.05)(12)} \\ P &= 6000\left(1+\frac{.05}{12}\right)^{-(.05)(12)} \\ P &\approx 5852.163 \end{aligned}$$

. Therefore, I should save \$5852.15 in this account in order to reach my target amount of money in 6 months.

5. A necklace is appraised at \$6300. If the value of the necklace has increased at an annual rate of 7%, how much was it worth 15 years ago?

The annual rate of interest is 7%, so we know that $n = 1$ time per year, and $r = .07$. Also, after $t = 15$ years, the value of the necklace ($A$) is 6300. Using $A = P\left(1+\frac{r}{n}\right)^{nt}$, we have

$$\begin{aligned} A &= P\left(1+\frac{r}{n}\right)^{nt} \\ 6300 &= P\left(1+\frac{.07}{1}\right)^{(1)(15)} \\ P &= 6300\left(1+\frac{.07}{1}\right)^{-(1)(15)} \\ P &\approx 2283.41. \end{aligned}$$

So, the necklace was originally worth \$2283.41.