

Erratum to: Differential Galois theory of linear difference equations

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Ruyong Feng pointed out that the proof of Proposition 2.9 contains a mistake and supplied a correct proof, which we present below.

In the second line of the proof, we say “*Therefore* $\partial(Z) - BZ = DZ$ for some $D \in \text{gl}_n(k^\sigma)$ ”. This is incorrect and the correct statement should be “*Therefore* $\partial(Z) - BZ = ZD$ for some $D \in \text{gl}_n(k^\sigma)$ ”. This change invalidates the argument in the first paragraph of that proof. The following is a replacement for the first paragraph of the proof. The remaining parts of the proof are correct.

Assume that such a B exists. A calculation shows that $\sigma(\partial(Z) - BZ) = A(\partial(Z) - BZ)$. Therefore $\partial(Z) - BZ = ZD$ for some $D \in \text{gl}_n(k^\sigma)$. Since k^σ is differentially closed, there is a $U \in \text{GL}_n(k^\sigma)$ such that $\partial(U) = -DU$. We then have

$$\partial(ZU) = \partial(Z)U + Z\partial(U) = (BZ + ZD)U - ZDU = B(ZU).$$

For any $\phi \in \text{Aut}_{\sigma\partial}(R/k)$, let $[\phi]_Z \in \text{GL}_n(k^\sigma)$ denote the matrix such that $\phi(Z) = Z[\phi]_Z$. We then have

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$$\begin{aligned}\partial(\phi(ZU)) &= \partial\left((ZU)U^{-1}[\phi]_ZU\right) = \partial(ZU)U^{-1}[\phi]_ZU + ZU\partial\left(U^{-1}[\phi]_ZU\right) \\ &= \phi(\partial(ZU)) = \phi(BZU) = BZUU^{-1}[\phi]_ZU,\end{aligned}$$

which implies $ZU\partial(U^{-1}[\phi]_ZU) = 0$. Therefore $\partial(U^{-1}[\phi]_ZU) = 0$ and so $U^{-1}[\phi]_ZU \in \text{GL}_n(\mathbb{C})$.

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