PRACTICE TEST II

Show all work.

1. (25 points)
   a) Sum the series \( \frac{2}{5} - \frac{2}{25} + \frac{2}{125} - \frac{2}{625} + \ldots \).
   b) A doctor wishes to give a patient a daily dose of D mgs of a certain medicine. The doctor knows that the body eliminates each day 80% of the amount present in the body. In the long run, the doctor would like the amount of drug in the body to approach 4 mgs. What should D be?

2. (25 points)
   a) Find the third Taylor Polynomial of \( \sqrt{x} \) at \( x = 4 \).
   b) Use a) to approximate \( \sqrt{3.5} \).

3. (25 points)
   a) Using the Taylor series \( \frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots \), find the first three nonzero terms of the Taylor series of \( \frac{x}{1 + x^2} \).
   b) Given that \( \cos x = 1 - x^2/2! + x^4/4! - x^6/6! + \ldots \), find the first three nonzero terms of an infinite series converging to

\[
\int_0^1 \frac{\cos(x) - 1}{x} \, dx
\]

4. (25 points)
   a) Evaluate

\[
\int_R \int x e^{xy} \, dy \, dx
\]

where R is the region bounded by \( x = 0, x = 1, y = 0, \) and \( y = 2 \).
   b) Evaluate

\[
\int_0^1 (\int_y^2 y \, x \, dx) \, dy
\]