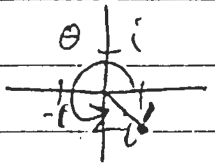
MA 303 HW #3

p. 27 #6

(a) $$(1-i)^{19} = \left(\sqrt{2}\, e^{i\frac{7\pi}{4}}\right)^{19}$$

$$= 2^{\frac{19}{2}}\, e^{i\frac{7\cdot 19\pi}{4}}$$



$|1-i| = \sqrt{2}$

$\tan\theta = -1$

$\theta = \frac{7\pi}{4}$

$$= 2^{9}\sqrt{2}\, e^{i\frac{133\pi}{4}}$$

$$= 512\sqrt{2}\, e^{i\left(32+\frac{5}{4}\right)\pi}$$

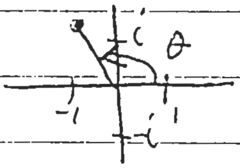
$$= 512\sqrt{2}\, e^{i\frac{5\pi}{4}}$$

$$= 512\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$$

$$= 512\sqrt{2}\left(-\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right) = -512-512i$$

(b) $$\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{37} = \left(1\, e^{i\frac{2\pi}{3}}\right)^{37}$$

$$= e^{i\frac{74\pi}{3}}$$



$\left|-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right| = \sqrt{\frac{1}{4}+\frac{3}{4}} = 1$

$\tan\theta = \dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$

$\theta = \frac{2\pi}{3}$

$$= e^{i\left(24+\frac{2}{3}\right)\pi}$$

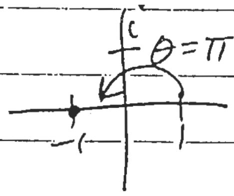
$$= e^{i\frac{2}{3}\pi}$$

$$= \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$$

$$= -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

(c) $$(-1)^{n} = \left(e^{i\pi}\right)^{n}$$ n an integer

$$= e^{in\pi}$$

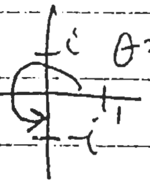


$$= \cos(n\pi) + i\sin(n\pi)$$

$$= \cos(n\pi)$$

(d) $$(-i)^{n} = \left(e^{i\frac{3\pi}{2}}\right)^{n}$$

$$= e^{i\frac{3n\pi}{2}}$$



$$= \cos\left(\frac{3\pi n}{2}\right) + i\sin\left(\frac{3\pi n}{2}\right)$$

Use $\frac{3\pi n}{2} = n\pi + \frac{n\pi}{2}$

$$= \left[\overbrace{\cos n\pi}^{=(-1)^n}\ \overbrace{\cos\frac{n\pi}{2}}^{=0 \text{ odd } n} - \overbrace{\sin n\pi}^{=0}\sin\frac{n\pi}{2}\right] + i\left[\overbrace{\sin n\pi}^{=0}\cos\frac{n\pi}{2} + \overbrace{\cos n\pi}^{=(-1)^n}\ \overbrace{\sin\frac{n\pi}{2}}^{=0\ n \text{ even}}\right]$$

$$= \cos\left(\frac{n\pi}{2}\right) - i\sin\left(\frac{n\pi}{2}\right)$$

$$\boxed{\begin{array}{l}\text{Use negative } \theta: \\ (-i)^{n} = \left(e^{-\frac{\pi}{2}i}\right)^{n} \\ \quad = \cos\left(\frac{n\pi}{2}\right) + i\sin\left(-\frac{n\pi}{2}\right) \\ \quad = \cos\frac{n\pi}{2} - i\sin\left(\frac{n\pi}{2}\right)\end{array}}$$

MA 303 HW #3

p.27 #10

(a) Solve $z^4 = -1 = e^{i(\pi + 2\pi k)}$

$$\Rightarrow \quad z = \left[e^{i(\pi+2\pi k)}\right]^{\frac{1}{4}} = e^{i\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)} \qquad k=0,1,2,3$$

$k=0 \qquad z_0 = e^{i\frac{\pi}{4}} = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

$k=1 \qquad z_1 = e^{i\left(\frac{\pi}{4}+\frac{\pi}{2}\right)} = \cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4} = -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

$k=2 \qquad z_2 = e^{i\left(\frac{\pi}{4}+\pi\right)} = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$

$k=3 \qquad z_3 = e^{i\left(\frac{\pi}{4}+\frac{3\pi}{2}\right)} = \cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4} = \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$

(c) Solve $z^2 = i = e^{i\left(\frac{\pi}{2}+2\pi k\right)}$

$$\Rightarrow \quad z = \left[e^{i\left(\frac{\pi}{2}+2\pi k\right)}\right]^{\frac{1}{2}} = e^{i\left(\frac{\pi}{4}+\pi k\right)} \qquad k=0,1$$

$k=0 \qquad z_0 = e^{i\frac{\pi}{4}} = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

$k=1 \qquad z_1 = e^{i\left(\frac{\pi}{4}+\pi\right)} = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$