HW #2

p. A19 #29 $\quad y''-4y'+3y = xe^{-x}$

ch. eq. $0 = r^2-4r+3 = (r-3)(r-1) \Rightarrow r = 3, 1$

$$\boxed{y_h(x) = c_1 e^{3x} + c_2 e^{x}}$$

Find y_p.

$g(x) = xe^{-x}$, $g'(x) = e^{-x} - xe^{-x}$, $g''(x) = -e^{-x} - e^{-x} + xe^{-x}$

So $y_p(x) = Ae^{-x} + Bxe^{-x}$

$y' = -Ae^{-x} + Be^{-x} - Bxe^{-x}$

$y'' = Ae^{-x} - Be^{-x} - Be^{-x} + Bxe^{-x}$

In (o.d.e.):

$$Ae^{-x} \boxed{-2Be^{-x}} + Bxe^{-x} + 4Ae^{-x} \boxed{-4Be^{-x}} + 4Bxe^{-x}$$
$$+ 3Ae^{-x} + 3Bxe^{-x} = xe^{-x}$$

$\Leftrightarrow 8Ae^{-x} - 6Be^{-x} + 8Bxe^{-x} = xe^{-x}$

$$\Rightarrow \begin{cases} 8A - 6B = 0 \\ 8B = 1 \end{cases} \Rightarrow \boxed{B = \tfrac{1}{8}} \text{ and } \boxed{A = \tfrac{3}{4}B = \tfrac{3}{32}}$$

Gen sol. $\boxed{y(x) = c_1 e^{3x} + c_2 e^{x} + \frac{3}{32} e^{-x} + \frac{1}{8} x e^{-x}}$

p. A26 #35 $\quad x^2y'' + xy' + 4y = 0$, $x>0$, Euler's Eq.

$y(x) = x^r$, $y' = rx^{r-1}$, $y'' = r(r-1)x^{r-2}$

In (o.d.e.):

$$x^2 r(r-1)x^{r-2} + xrx^{r-1} + 4x^r = 0$$

$\Leftrightarrow \quad x^r[r^2 - r + r + 4] = 0$

$\Rightarrow \quad r^2 + 4 = 0 \quad \Rightarrow \quad r = \pm 2i$

$\rightarrow$ Complex sol. $\quad z(x) = x^{2i} = e^{2i\ln x}$

$\rightarrow z(x) = \cos(2\ln x) + i\sin(2\ln x)$

Gen sol. $\quad \boxed{y(x) = c_1 \cos(2\ln x) + c_2 \sin(2\ln x)}$

HW #2

p.331 #9 $(1, x) = \int_{-1}^{1} 1 \cdot x\,dx = \frac{1}{2}x^2\Big]_{-1}^{1} = 0$

$0 = (1, a + bx + x^2) = \int_{-1}^{1} (a + bx + x^2)\,dx = 2a + \frac{2}{3} \Rightarrow \boxed{a = -\frac{1}{3}}$

$0 = (x, a + bx + x^2) = \int_{-1}^{1} (ax + bx^2 + x^3)\,dx = \frac{a}{2} - \frac{a}{2} + \frac{2b}{3} \Rightarrow \boxed{b = 0}$

p.344 #13 $y'' + \lambda y = 0$ B.C. $y(0) = 0$ and $y'(\pi) = 0$

ch. eq $r^2 + \lambda = 0$

case 1: $\lambda < 0$. Take $\lambda = -\omega^2$ $\omega > 0$.

$y(x) = c_1 \cosh \omega x + c_2 \sinh \omega x$ and $y'(x) = c_1 \omega \sinh \omega x + c_2 \omega \cosh \omega x$

At $x = 0$ $0 = y(0) = c_1$

At $x = \pi$ $0 = y'(\pi) = c_2 \underbrace{\omega \cosh \omega\pi}_{\neq 0} \Rightarrow c_2 = 0$ $\boxed{\text{Trivial Sol.}}$

case 2: $\lambda = 0$, $y(x) = c_1 x + c_2$ $y'(x) = c_1$

At $x = 0$ $0 = y(0) = c_2$

At $x = \pi$ $0 = y'(\pi) = c_1$ $\boxed{\text{Trivial Sol.}}$

case 3: $\lambda > 0$. Take $\lambda = \omega^2$ $\omega > 0$.

$y(x) = c_1 \cos \omega x + c_2 \sin \omega x$ $y'(x) = -c_1 \omega \sin \omega x + c_2 \omega \cos \omega x$

At $x = 0$ $0 = y(0) = c_1$

At $x = \pi$ $0 = y'(\pi) = c_2 \omega \cos(\omega\pi)$ For nontrivial sol., need $\cos \omega\pi = 0$

Thus $\omega\pi = (\text{odd})\frac{\pi}{2} \Rightarrow \omega_n = \frac{2n+1}{2}$ for $n = 0, 1, 2, 3, \ldots$

Thus $\boxed{\lambda_n = \left(\frac{2n+1}{2}\right)^2}$ and $\boxed{y_n = \sin\left(\frac{(2n+1)x}{2}\right)}$ $n = 0, 1, 2, 3 \ldots$