HW #1

p. A8 #2 $\quad y' + 2xy = x$

$I.F. = e^{\int 2x\,dx} = e^{x^2}$

$e^{x^2} y' + 2x e^{x^2} y = x e^{x^2}$

$\int \frac{d}{dx}\left(y e^{x^2}\right) dx = \int x e^{x^2}\, dx$

$u = x^2 \qquad du = 2x\,dx$

$\Rightarrow \; y e^{x^2} = \frac{1}{2}\int e^u\, du = \frac{1}{2} e^{x^2} + c$

$\Rightarrow \; \boxed{y = \frac{1}{2} + c e^{-x^2}}$

p. A26 #11 $\quad x^2 y'' + x y' - y = 0 \quad , \quad y_1 = x$ ✓

y_1 in (o.d.e): $\quad x^2 y_1'' + x y_1' - y_1 = 0 + x \cdot 1 - x = 0$

To find y_2 using reduction of order formula, write (o.d.e.) as:

$y'' + \frac{1}{x} y' - \frac{1}{x^2} y = 0$.

OR Assume $y_2(x) = v(x)\, y_1(x) = v \cdot x$.

$\Rightarrow y_2' = v'x + v \; , \; y_2'' = v''x + v' + v'$

In (o.d.e.):

$x^3 v'' + 2x^2 v' + x^2 v' + xv - vx = 0$

$\Rightarrow \quad x v'' + 3v' = 0$

Let $w = v'$ then $\quad x w' = -3w$ and separate variables

$\Rightarrow \int \frac{1}{w}\, dw = \int -\frac{3}{x}\, dx$

$\Rightarrow \ln w = -3 \ln x$

$\Rightarrow v' = w = x^{-3}$

$\Rightarrow \boxed{v = \int x^{-3}\, dx = -\frac{1}{2} x^{-2}}$

Thus $y_2(x) = -\frac{1}{2} x^{-2} \cdot x = -\frac{1}{2} x^{-1}$

Gen sol. is $\quad \boxed{y(x) = c_1 x + c_2 x^{-1}}$