MA 401 HW #1

p. A19 #29

$$y'' - 4y' + 3y = xe^{-x}$$

ch. eq. $\lambda^2 - 4\lambda + 3 = 0 \iff (\lambda-3)(\lambda-1) = 0 \Rightarrow \lambda = 3, 1$

$$\boxed{y_h(x) = c_1 e^{3x} + c_2 e^{x}}$$

$g(x) = xe^{-x}$ so $g'(x) = e^{-x} + xe^{-x}(-1)$

Thus need $\{e^{-x}, xe^{-x}\}$ for $y_p(x)$.

Assume $\boxed{y_p(x) = Axe^{-x} + Be^{-x}}$

$$y' = Ae^{-x} - Axe^{-x} - Be^{-x}$$

$$y'' = (A-B)e^{-x}(-1) - Ae^{-x} + Axe^{-x}.$$

In (ode):

$$(B-2A)e^{-x} + Axe^{-x} - 4\left((A-B)e^{-x} - Axe^{-x}\right) + 3Axe^{-x} + 3Be^{-x} = xe^{-x}$$

$$\Rightarrow (B - 2A - 4A + 4B + 3B)e^{-x} + (A + 4A + 3A)xe^{-x} = xe^{-x}$$

$$\Rightarrow \begin{cases} 8A = 1 \\ 8B - 6A = 0 \end{cases} \Rightarrow \boxed{\begin{array}{l} A = \frac{1}{8} \\ B = \frac{6}{64} = \frac{3}{32} \end{array}}$$

Gen. sol. $\boxed{y(x) = c_1 e^{3x} + c_2 e^{x} + \frac{1}{8}xe^{-x} + \frac{3}{32}e^{-x}}$