

Name: _____

Use of books, notes or calculators is **NOT** permitted.

Please show all your work! Answers without appropriate supporting work may not receive full credit.

Clearly indicate your answers to each problem by underlining them or placing a box around your answers!

Trigonometric functions at the values $0, \pi/6, \pi/4, \pi/3, \pi/2$, etc must be evaluated!

T/F Questions are graded with NO PARTIAL CREDIT.

Exam Score

Problem	Score	Out of:
1		10
2		20
3		20
4		10
5		20
6		20
7		5
Total		105

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1. [10] For the True/False questions below, clearly circle your answer.

T or F If f and g are increasing functions on an interval I , then $f - g$ is increasing on I .

T or F If f and g are increasing functions on an interval I then fg is increasing on I .

T or F If f is increasing and $f(x) > 0$ on I , then $g(x) = 1/f(x)$ is decreasing on I .

T or F If f has an absolute minimum value at c , then $f'(c) = 0$.

T or F There exists a function f such that $f(x) > 0$, $f'(x) < 0$, and $f''(x) > 0$ for all x .

2. [20] A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 4 feet per second. How fast is the top moving down when the base of the ladder is 15 feet from the wall?

3. [20] Find the following limits. Verify if L'Hospital Rule applies, before using it.

(a) [5 pts]

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{e^x}$$

(b) [5 pts]

$$\lim_{x \rightarrow 0^+} \sqrt{x} \ln x$$

(c) [5 pts]

$$\lim_{x \rightarrow \infty} x^2 e^{-x^2}$$

(d) [5 pts]

$$\lim_{x \rightarrow \infty} (1 + 2x)^{\frac{1}{x}}$$

4. [10] Use a linear approximation to estimate $(2.01)^5$
5. [20] A rectangular page is to contain 24 square inches of print. The margins at the top and bottom of the page are to be $1\frac{1}{2}$ inches, and the margins on the left and right are to be 1 inch. What should the dimensions of the page be so that the least amount of paper is used?
- (a) [3 pts] Draw a picture of the rectangular page and its printed area. Label with x and y the dimensions of the printed area.
- (b) [3 pts] Express the area to be minimized as a function of x and y .
- (c) [3 pts] Express the area of print as a function of x and y .

(d) [3 pts] Express the area to be minimized as a function of x *only*. Use the equations you found in (b) and (c) to do this.

(e) [8 pts] Find the dimensions of the page which minimize the amount of paper used. Place your answers (length, width) in a box.

6. [20] Let us consider the function $f(x) = x^3 - 3x^2 + 1$ on the closed interval $[-1, 4]$.
- (a) [10 pts] Find the absolute maximum and minimum values of f and state where those values occur.
- (b) [10 pts] On what subinterval of $[-1, 4]$ is f increasing/decreasing? On what subinterval of $[-1, 4]$ is f concave up/down? Sketch the graph of f .

7. [**5 Bonus Points**] Calculate up to four decimal places x_3 , the third iterate of Newton's Method which approximates a zero of $f(x) = x^2 - 2$. Use $x_1 = 1$ as initial guess. You may use the fact that $0.25/3 = 0.0833$.